

### Math 105: Review for Final Exam, Part II

1. Your company is mass-producing a cylindrical container. The flat portion (top and bottom) costs 3 cents per square inch and the curved (lateral) portion costs 5 cents per square inch. If your budget is \$9.00 per container, what dimensions will give the largest volume?

$$\text{area of circle} = \pi r^2 \qquad \text{lateral area of cylinder} = 2\pi r h \qquad \text{volume of cylinder} = \pi r^2 h$$

2. You are watching a plane flying toward your position at a constant height of 3 miles and a speed of 500 miles per hour relative to the ground. At the moment when the plane is 5 miles from you (diagonally), at what rate is the angle of your vision toward the plane changing?

3. Use the Intermediate Value Theorem to show that  $f(x) = x^3 - 2x - 1$  has a root on  $[1, 2]$ .

4. What (if anything) does the Extreme Value Theorem say about  $f(x) = x^2$  on each of the following intervals?

(a)  $[1, 4]$

(b)  $(1, 4)$

5. Find the value of the constant  $c$  that the Mean Value Theorem specifies for  $f(x) = x^3 + x$  on  $[0, 3]$ .

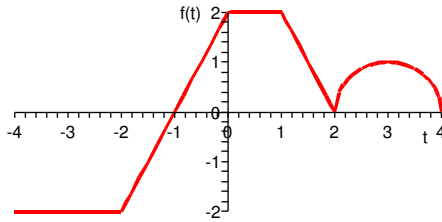
6. Water is leaking out of a tank at a decreasing rate  $r(t)$  as shown in the table below.

time (min)	0	2	4	6	8
rate (gal/min)	15	11	8	4	3

(a) Find an overestimate and underestimate for the total amount that leaked out during these 8 minutes.

(b) Interpret the expression  $\int_2^6 r(t) dt$  in terms of the situation described above.

7. Consider the graph of  $f(t)$  shown. It is made of straight lines and a semicircle.



Let  $G(x) = \int_0^x f(t) dt$  and  $H(x) = \int_{-3}^x f(t) dt$ .

- (a) Compute  $G(2)$ ,  $G(4)$ ,  $G(-4)$ , and  $H(4)$ .
- (b) Where is  $G$  increasing? Where is  $G$  decreasing?
- (c) Where is  $G$  concave up? Where is  $G$  concave down?
- (d) At what  $x$ -value(s) does  $G$  have a local maximum? At what  $x$ -value(s) does  $G$  have a local minimum?
- (e) Find a formula that relates  $G$  and  $H$ .
- (f) How would your answers to (b), (c), and (d) change if the questions were about  $H$  instead of  $G$ ?
8. (a) Use sigma notation to express  $L_{10}$  and  $M_{10}$  as approximations to  $\int_{20}^{60} \ln x dx$ .  
 [Students in the 9:30 section should omit this problem.]
- (b) Draw a sketch that represents the sum  $M_4$ .  
 [Students in the 9:30 section should omit this problem.]

9. Find the following.

(a) all antiderivatives of  $1 + 2x + x^3 + 4\sqrt{x} + \frac{1}{x^5} + \sec^2(6x)$

(b)  $\int_{-2}^2 \sqrt{4-x^2} dx$

(c)  $\frac{d}{dx} \int_1^x \sin \sqrt{t} dt$

(d)  $\int_0^2 x^2 dx$

[Students in the 9:30 section should omit this problem.]

Do this first with the limit definition of the definite integral then check your answer with the Fundamental Theorem.

You may use the fact that  $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$ .