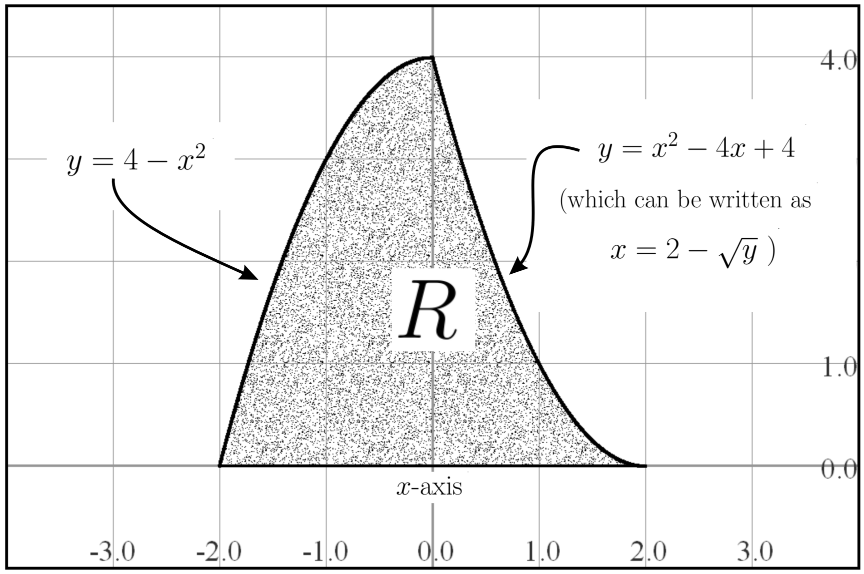
1. Let R be the region shown in the figure to the right; the curves forming its boundaries are labeled.

1A. Find the area of R by setting up and evaluating the appropriate integral(s). Show all your steps.



1B. Suppose the region R is revolved around the line $x = 3$. Set up the integral(s) which represents the volume of the resulting solid. You do *not* need to evaluate your integral(s).

2A. Find the exact value (call it "EV") of $\int_0^{1.5} x \cos x^2 \, dx$ by using a substitution and the Fundamental Theorem of Calculus. Show all the steps, give the exact value *and* also express EV as a decimal number to as many places as your calculator displays. (For example, if you get $\sqrt{17}$ for EV you would also list the decimal number 4.123105626)

2B. What are the limits on the integral in (2A) after the required substitution was made?

2C. Find MID(100), the midpoint rule approximation of $\int_0^{1.5} x \cos x^2 \, dx$ if 100 subdivisions are used.

2D. What does theorem 3 of chapter 6 guarantee is the worst possible error MID(100) can give for the integral in 2A?
Two hints: (1) if $f(x) = x\cos(x^2)$ then $f''(x) = -6\,x\sin(x^2) - 4\,x^3\cos(x^2)$;
(2) a nice window for plotting f'' is $[0,\ 1.5] \times [-10,\ 5]$.
NOTE: Choose the best K_2 to *one* digit after the decimal point. Give your ANSWER to *eight* places after the decimal point, NOT in scientific notation!

2E. What is (to *eight* places after the decimal point) the exact error in using MID(100) as an approximation to EV?

3A. From memory, what is the Maclaurin series for $\cos t$? Explicitly write the series through the first five non-zero terms.

3B. Use the correct answer to 3A to find the first five non-zero terms of the Maclaurin series for $x\cos(x^2)$.

3C. Use the correct answer to 3B to find the first five non-zero terms of the Maclaurin series series for $\int x\cos(x^2)\,dx$, that is, a series for an antiderivative of $x\cos(x^2)$. (You can let the constant C of integration be 0).

3D. Use just the first *four* terms of the answer to 3C to estimate $\int_0^{1.5} x\cos x^2\,dx$. How does the result compare to the exact value (you found it in (2A), previous problem).

3E. *Bonus!* Let $f(x) = x\cos(x^2)$. What is $f^{(9)}(0)$? Show your reasoning.

4. Find $\int \frac{1}{x^2\sqrt{x^2-9}}\,dx$ by making the substitution $x = 3\sec t$. Show all relevant triangles!

5. Find $\int \arctan(Ax)\,dx$ using "LIATE" and integration by parts. Here A is a constant.

6. Consider the power series $s(x) = \sum_{k=1}^{\infty} \frac{9\, k^4\, (x-5)^k}{2^{k+3}}$.

6A. Use the ratio test to find the interval of convergence (IOC) for this series (with the endpoints pending).

6B. Determine which, if either, endpoint belongs to the IOC, and explain your reasons.

7. Consider the alternating series $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{\sqrt[3]{k^2}}$.

7A) Explicitly write out the partial sum $\displaystyle\sum_{k=2}^{5} \frac{(-1)^k}{\sqrt[3]{k^2}}$. (No decimals, just write them with radicals)

7B) Explain why the series $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{\sqrt[3]{k^2}}$ passes the alternating series test (AST), and therefore converges to some number M. (You do not need to find M).

7C) Find the partial sum $\displaystyle\sum_{k=2}^{100} \frac{(-1)^k}{\sqrt[3]{k^2}}$. Tell me values you used for B, A, and N in the LHS program.

7D) Find the smallest N for which the partial sum $\displaystyle\sum_{k=2}^{N} \frac{(-1)^k}{\sqrt[3]{k^2}}$ is within $\epsilon = 1/1000$ of the value M to which the series converges. Hint: remember that if an alternating series converges to M, then $|M - \displaystyle\sum_{k=2}^{N}(-1)^k\, c_k| \, < \, c_{N+1}$.

This problem continues, NEXT PAGE!

7, continued:

7E) Show all the steps involved in determining if $\int_2^\infty x^{-2/3}\, dx$ converges, and if so to what.

7F) Is the series $\sum_{k=2}^{\infty} \frac{(-1)^k}{\sqrt[3]{k^2}}$ *absolutely* convergent? Explain your answer. The integral test may be useful.

7G) *BONUS:* How many days would it take your calculator to find the sum in 7D, based on how long it took to do the sum in 7C (I'll need you to tell me how long your calculator took, in seconds, to do 7C; just time it roughly).

8A. Explain why $f(x) = \dfrac{1-\cos(2\pi x)}{10000}$ is a probability density function on the interval $[0, 10000]$. Show all your steps in the computations of any necessary integrals.

[*June 2013 EDIT!* NOTE: The function given above, and done by students at exam time, was a TYPO; I meant to let $f(x) = \dfrac{1-\cos(2\pi x/10000)}{10000}$. Both functions are pdf's, but that first one is rather unrealistic — graph both and compare to see why! The suggested solutions file contains solutions to BOTH problems].

8B. Suppose f represents the distribution of how many pictures a certain model of camera take before the camera breaks. What's the probability that if you buy such a camera, it fails before you take 2500 pictures?

9. Set up the partial fraction decomposition for $\int \frac{3x^2+2x+1}{(x^2-16)(x^2+16)}\,dx$. You do NOT have to find the values of the A's, B's, C's, and so on in your answer, and you do NOT have to do any integrals.

10. Find the value of the series $64 - 16 + 4 - 1 + 1/4 - 1/16 + \cdots$

Bonus: Find a general formula for the sum of the first N terms in this series.

11A. Either from memory or by cooking it up right now, what is the Maclaurin series, and the IOC, for $\arctan x$? (Give the first five non-zero terms).

11B. Either from memory or by cooking it up right now, what is the Maclaurin series, and the IOC, for e^x? (Give the first five non-zero terms).