

FINAL

Math 105
4/10/12

Name: _____
by writing my name I swear this work is my own

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 11 problems and is worth 100 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (12 points) The following is a graph of f' on the interval $[-1,3]$. Assume open circles at all jump discontinuities.

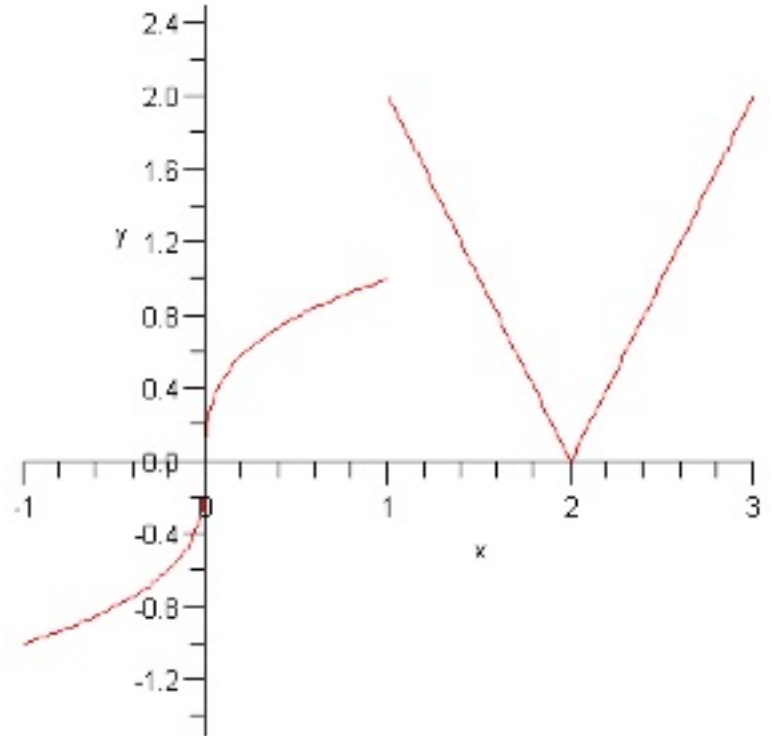
a. (2 pts) Where is $f(x)$ increasing?

b. (2 pts) Where is $f(x)$ decreasing?

c. (3 pts) Where is $f(x)$ concave up?

d. (3 pts) Where is $f(x)$ concave down?

e. (2 pts) Where is $f'(x)$ **NOT** differentiable?



2. (10 points)

x	$f(x)$	$g(x)$	$j(x)$	$f'(x)$	$g'(x)$	$j'(x)$
-2	0	1	-1	3	2	1
-1	1	3	2	-1	3	-2
0	2	1	1	2	-2	2
1	3	1	-1	0	3	1
2	-2	2	1	3	0	3
3	-1	1	-1	1	-2	0

a. (5 pts) $H(x) = 2f(g(x)) + e^{j(x)}$. Find $H'(0)$.

b. (5 pts) $F(x) = \frac{x^2 j(x)}{f(x)^3}$. Find $F'(1)$.

3. (12 points) Find the derivative of the following functions.

a. (4 pts) $f(s) = \sqrt[4]{s^3} + \frac{4}{s^2} + \ln(s) + \pi$

b. (4 pts) $f(x) = \frac{2e^{6x+3} + x^3}{\arcsin(3x)} + \ln(3x^2 + x - 1)$

c. (4 pts) $y = \frac{(2x + 4)^3(x^2 - 2)^3}{(x - 4)^4}$ (Use logarithmic differentiation)

4. (6 points) For the equation $x^3 + y^3 + 2xy = (x - y)^2$ use implicit differentiation to find $\frac{dy}{dx}$.

5. (14 points) Evaluate the following limits. Only use L'Hôpital's rule when appropriate. Show your work!!

a. (3 pts) $\lim_{x \rightarrow 0} \frac{\cos(x)}{x^2}$

b. (3 pts) $\lim_{x \rightarrow \infty} \frac{4x^3 + 2x^2 - x - 3}{5x^3 - 3x + 7}$

c. (4 pts) $\lim_{x \rightarrow 0^+} \frac{e^x - e}{\ln x}$

d. (4 pts) $\lim_{x \rightarrow 1} \frac{e^x - e}{\ln x}$

6. (8 points) Choose **one** of the following.

1. A rocket that is launched vertically is tracked by a radar station on the ground 5 miles from the launch site. If the distance from the rocket to the radar station is increasing at a rate of 3000 miles per hour, find the vertical speed of the rocket when its distance from the radar station is 13 miles?
2. Tom stands 2 miles directly north of Sally. If Tom walks straight north and Sally walks straight west, both at a speed of 3 mph, how fast is the distance between Tom and Sally increasing when they've walked for 2 hours?

7. (*10 points*) Choose **one** of the following

1. A Norman window has the shape of a rectangle topped by a semicircle. If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is admitted (ie, area of glass is maximized).
2. A closed rectangular display box is to be built with the following criteria. The length of the base is twice the width of the base. The base and the top of the box are to be made of glass that costs 3 dollars per square foot. The other sides are made of wood which costs 2 dollar per square foot. What are the dimensions of the box of greatest volume that can be constructed for 72 dollars?

8. (4 points) Verify that $y = \cos(4x) + 1$ solves following initial value problem.

$$y'' + 16y = 16, y(0) = 2$$

9. (4 points) Let $\int_0^3 f(x)dx = 10$, $\int_3^5 f(x)dx = 9$ and $\int_2^4 f(x)dx = 4$. Find the following.

a. (2 pts) $\int_0^5 f(x)dx$

b. (2 pts) $\int_5^3 f(x)dx$.

10. (8 points)

a. (5 pts) Determine the antiderivative of $t^2 + \frac{2t}{1 + 4t^2}$. Show your check.

b. (3 pts) Use the FTC to determine $\int_0^2 t^2 + \frac{2t}{1 + 4t^2} dt$.

11. (12 points) Let $f(x) = 2x - 4$.

a. (2 pts) Draw the graph of the function and determine $\int_1^4 f(x)$ using geometry (the area of basic shapes).

b. (2 pts) Determine $\int_1^4 f(x)$ using the FTC.

c. (8 pts) Use infinite Riemann sums (limit as n approaches infinity of the sum) to find the area under the curve $f(x) = 2x - 4$ for x from 1 to 4 $\left(\int_1^4 f(x)\right)$. You will need the fact that $\sum_{i=1}^n i = \frac{n(n+1)}{2}$.