FINAL

Math 105
4/8/14

Name: $\underbrace{\hspace{6cm}}$
by writing my name I swear this work is my own

Read all of the following information before starting the exam:

- Put all of your work in the blue book EXCEPT for the last problem (do that on the test sheet). Turn in the blue book and test sheet.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 10 problems and is worth 100 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!
- Some useful formulas:

  Area of a circle with radius r, $A = \pi r^2$,

  Circumference of a circle with radius r, $C = 2\pi r$.

  Area of a rectangle with sides x and y, $A = xy$,

  Perimeter of a rectangle with sides x, y, $P = 2x + 2y$

  Volume of a rectangular prism with sides x, y, z, $V = xyz$,

  Surface area of rectangular prism with x, y, z, $SA = 2xy + 2yz + 2xz$

  Volume of a cylinder with radius r and height h, $V = \pi r^2 h$,

  Surface area of a cylinder with radius r and height h, $SA = 2\pi r^2 + 2\pi rh$

  $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

  $$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

  $$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

1. (*6 points*) Sketch a possible graph of a single function f that satisfies all the following conditions:

1. $f'(x) > 0$ on $(-\infty, 1)$, $f'(x) < 0$ on $(1, \infty)$
2. $f''(x) > 0$ on $(-\infty, -2)$ and $(2, \infty)$, $f''(x) < 0$ on $(-2, 2)$
3. $\lim_{x\to-\infty} f(x) = -2$, $\lim_{x\to\infty} f(x) = 0$

2. (*19 points*) Find the derivative using the appropriate rules.

a. (*6 pts*) $h(x) = (\cos^3(x) + 3\cos(x) + 7)^9 + \frac{\sqrt{2x+1}}{x+2} + 2e^{-x^2/2}$

b. (*5 pts*) $y = \dfrac{\sin^4(x)e^{3x}}{(4x^4 - 3x^2 - 2)^{15}}$ using logarithmic differentiation.

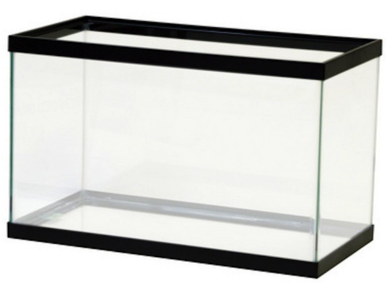
Use the following table to find the derivatives of the functions at the given values of x.

x	1	2	4
$f(x)$	4	2	6
$f'(x)$	5	7	4
$g(x)$	4	1	6
$g'(x)$	5	$\frac{1}{2}$	3

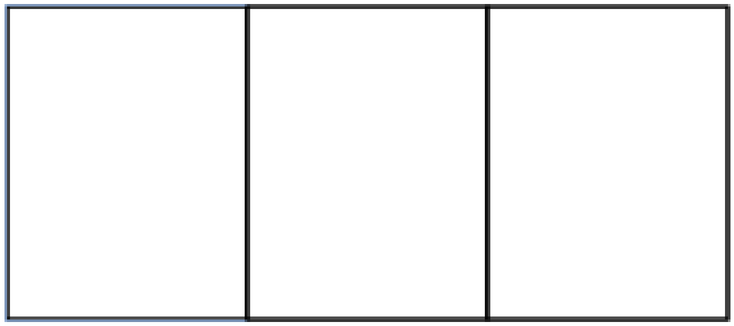
c. (*4 pts*) $g(\sqrt{x})f(x)$ at $x = 4$.

d. (*4 pts*) $f(g(x) - 2x)$ at $x = 1$.

3. (*8 points*) Water pours into a fish tank (standard rectangular prism) at a rate of 3 ft^3/min. How fast is the water level rising if the base of the tank is a rectangle with dimensions 2 ft $\times$ 3 ft?



4. (*10 points*) A warehouse consists of three separate spaces of equal size. Assume that the wall materials cost \$200 per linear foot and the company has allocated \$2,400,000 for the project.



a. (*8 pts*) What dimensions maximize the total area of the warehouse?

b. (*2 pts*) What is the area of each compartment in this case?

5. (*12 points*) Use L'Hopital's Rule if applicable. If not, use any other algebraic method learned this semester. You may certainly check your answer with a table.

a. (*4 pts*) $\lim_{x \to 16} \frac{\sqrt{x}+4}{x-16}$

b. (*4 pts*) $\lim_{x \to 0} \frac{e^x - x - 1}{\cos x - 1}$

c. (*4 pts*) $\lim_{x \to \infty} x^{1/x}$

6. (*8 points*) Let y be a function of x. Use implicit differentiation to find the an equation of the tangent line at the point (1,1) on the curve

$$y^4 + xy = x^3 - x + 2$$

7. (*7 points*) Determine the antiderivative of $e^{6t} + \frac{2t^3}{2+8t^4} + \frac{2}{2+8t^2}$.

8. (*8 points*) Verify the applicability of the IVT in the indicated interval for the give value. IF APPLICABLE find a value of c guaranteed by the theorem. IF NOT APPLICABLE, explain.

a. (*4 pts*) $f(x) = x^2 + x - 1, [-2, 5], f(c) = 11$

b. (*4 pts*) $f(x) = x^2 + x - 1, [-2, 5], f(c) = 0$

9. (*14 points*) Let $f(x) = 3x^2 - 4x$ on [0,3].

a. (*2 pts*) Find L_3.

b. (*4 pts*) Determine $\int_0^3 (3x^2 - 4x)dx$ using the FTC.

c. (*8 pts*) Use infinite Riemann sums to find the area under the curve $f(x) = 3x^2 - 4x$ on [0,3].

10. (*8 points*) **WRITE YOUR ANSWERS IN THE BLANKS. HAND IN THIS SHEET WITH YOUR EXAM.** Let F be an antiderivative of f. Consider the following proof that $F(b)-F(a) = \int_a^b f(x)dx$. Fill in the blanks. (**) should be the same entry.

$$F(b) - F(b)$$
$$=F(x_n) - F(x_0)$$

Using telescoping sums, $F(x_n) - F(x_0)$ can be written as

____.

We can write this in summation notation,

$$(**) \sum_{i=1}^{n} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .$$

Since $F(x)$ is continuous and bounded on $[a, b]$ and differentiable on (a, b), the MVT guarantees that there exists an x_i^* in $[x_{i-1}, x_i]$ such that

____.

which we can rewrite as

____.

Then by substitution,

$$(**) \sum_{i=1}^{n} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \sum_{i=1}^{n} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Since,

____.

____.

the sum is the same as $\sum_{i=1}^{n} f(x_i^*)\Delta x$.

If we take the limit as n approaches ∞ of both sides, then

$$F(b) - F(a) = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)dx.$$