

**Math 106: Review for Final Exam, Part I**

1. Find the following. [See Review for Exam II for integration tips and strategies.]

(a)  $\int 12x^2 \cos(x^3) dx$

(b)  $\int_0^{\infty} xe^{-3x} dx$

(c)  $\int_0^6 \frac{dx}{(x-4)^2}$

(d)  $\int \frac{3x^2 + 2x - 5}{(x^2 + 1)(x - 4)} dx$

(e)  $\int_0^{\pi/3} \tan^3 x \sec^5 x dx$

(f)  $\int \sqrt{25 - x^2} dx$

2. Find the best possible left, right, midpoint, trapezoidal, and Simpson's approximations to  $\int_{-2}^0 f(x) dx$  given the data in the table below.

$x$	-2	-1.5	-1	-0.5	0
$f(x)$	2	3	6	10	11

3. If you use numerical integration to estimate  $\int_a^b \ln x dx$  (where  $a$  and  $b$  are positive), how would the following be ordered from least to greatest?  $L_{100}$ ,  $R_{100}$ ,  $M_{100}$ ,  $T_{100}$ ,  $\int_a^b \ln x dx$ .

4. Find bounds for each of the following errors if  $I = \int_0^2 e^{-5x} dx$ .

(a)  $|I - R_{100}|$

(b)  $|I - T_{100}|$

(c)  $|I - M_{100}|$

5. If  $I = \int_0^2 e^{-5x} dx$ , how many subdivisions are required to obtain a midpoint sum approximation with error of at most  $1/1,000,000$ ?

6. Write an integral equal to the area between  $y = 2x + 3$  and  $y = x^2 + 7x - 3$ .

7. Compute the arc length of  $y = \sqrt{1 - x^2}$  from  $x = 0$  to  $x = 1/2$ .

8. Consider the region bounded by  $y = 0$ ,  $x = 2$ , and  $y = x^2$ . Write an integral equal to the volume of the object created when the region is revolved about

(a) the  $x$ -axis

(b) the line  $x = 5$

9. The probability density function (pdf) of the weights of newborn toads in a certain pond is given by  $f(x) = \frac{k}{(x+1)^4}$ , where  $x$  is the weight (in ounces). Note that the domain is  $x \geq 0$  since no toad can have a negative weight.

(a) What must be the value of  $k$ ?

(b) What fraction of the newborn toads weigh more than one ounce?

10. Find the solution to  $\frac{dy}{dx} = \frac{\cos x}{y^2}$  that passes through  $(0, 2)$ . [Students in the 9:30 section should omit this problem.]