

**Math 105: Review for Final Exam, Part I**

1. Consider the function  $f(x) = \frac{3}{5 - 2x}$ .

(a) Is this function continuous on the interval  $(-\infty, \infty)$ ? Explain.

(b) Compute the average rate of change of  $f$  on  $[2, 2.01]$ .

(c) Using the limit definition of the derivative, compute  $f'(x)$ .

(d) Find the equation of the tangent line to  $f$  at  $x = 2$ .

2. Given that  $f(0) = 2$ ,  $g(0) = 3$ ,  $f'(0) = 5$ ,  $g'(0) = 7$ , and  $f'(3) = \pi$  compute the following.

(a)  $h'(0)$  if  $h(x) = f(x)g(x)$

(b)  $j'(0)$  if  $j(x) = \frac{f(x)}{g(x)}$

(c)  $k'(0)$  if  $k(x) = f(g(x))$

3. Compute  $dy/dx$  for each of the following.

(a)  $y = x^5 + 5^x + e^5 + \frac{x}{5} + \frac{5}{x} + \frac{5}{\sqrt[5]{x}} + \ln(5x) + \arctan(5x) + \ln(5) + \sin 5$

(b)  $y = \sqrt[3]{x} \cos(7x^3)$

(c)  $y = \frac{e^x + e^\pi}{\tan 4 - 7x}$

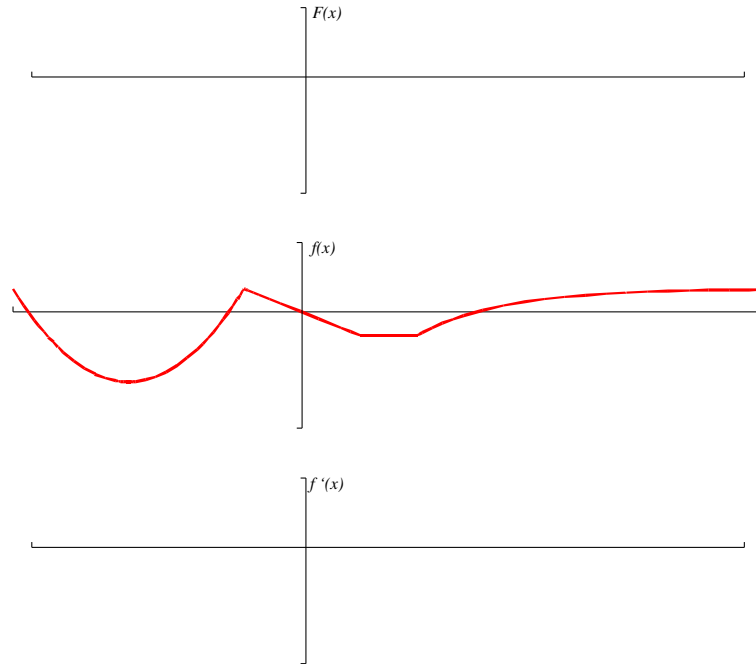
(d)  $y = \tan(e^{x^2} \arcsin(5x))$

(e)  $y^3 + yx^2 + x^2 = 3y^2$

(f)  $y = (x^2 + 1)^{\sin x}$

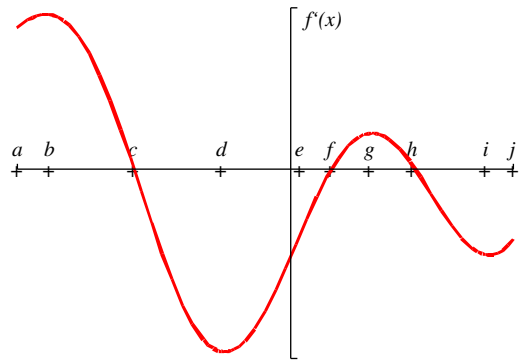
[Students in the 1:10 section may consider this as a bonus problem.]

4. Given the graph of  $f$ , sketch a graph of  $f'$  and a graph of  $F$ , an antiderivative of  $f$  such that  $F(0) = -1$ .



5. Shown below is a graph of  $f'$  on its entire domain. The graph is NOT  $f$ .

- At which  $x$ -value(s)
- (a) does  $f$  have a stationary point?
  - (b) does  $f$  have a local max?
  - (c) does  $f$  have a local min?
  - (d) does  $f'$  have a stationary point?
  - (e) does  $f'$  have a local max?
  - (f) does  $f'$  have a local min?
  - (g) is  $f$  greatest?
  - (h) is  $f$  least?
  - (i) is  $f'$  greatest?
  - (j) is  $f'$  least?
  - (k) is  $f''$  greatest?
  - (l) is  $f''$  least?
- (b)  $f$  decreasing?
  - (c)  $f'$  increasing?
  - (d)  $f'$  decreasing?
  - (e)  $f$  concave up?
  - (f)  $f$  concave down?



- On what interval(s) is
- (a)  $f$  increasing?

6. Is  $y = 7e^{3x}$  a solution to the differential equation  $y'' + 2y' - 15y = 0$ ? Explain.

7. Rewrite  $\sin(\arctan(5x))$  as an algebraic expression. [Students in the 8:00 section may omit this problem.]

8. Evaluate the following limits.

(a)  $\lim_{x \rightarrow \infty} \frac{x^2}{\ln x}$

(b)  $\lim_{x \rightarrow 0} \frac{\sin(12x) - 12x}{x^3}$

(c)  $\lim_{x \rightarrow 0} \frac{e^x - 1}{\cos x}$

(d)  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$