

Math 105: Review for Final Exam, Part I

1. Consider the function $f(x) = \frac{3}{5 - 2x}$.

(a) Is this function continuous on the interval $(-\infty, \infty)$? Explain.

(b) Compute the average rate of change of f on $[2, 2.01]$.

(c) Using the limit definition of the derivative, compute $f'(x)$.

(d) Find the equation of the tangent line to f at $x = 2$.

2. Given that $f(0) = 2$, $g(0) = 3$, $f'(0) = 5$, $g'(0) = 7$, and $f'(3) = \pi$ compute the following.

(a) $h'(0)$ if $h(z) = f(z)g(z)$

(b) $j'(0)$ if $j(z) = \frac{f(z)}{g(z)}$

(c) $k'(0)$ if $k(z) = f(g(z))$

3. (a) Find $\frac{dy}{dt}$ if $y = t^5 + 5^t + e^5 + \frac{t}{5} + \frac{5}{t} + \frac{5}{\sqrt[5]{t}} + \ln(5t) + \arctan(5t) + \ln(5) + \sin 5$.

(b) Find $\frac{dy}{dx}$ if $y = \sqrt[3]{x} \cos(7x^3)$.

(c) Find $\frac{dy}{dz}$ if $y = \frac{e^z + e^\pi}{\tan 4 - 7z}$.

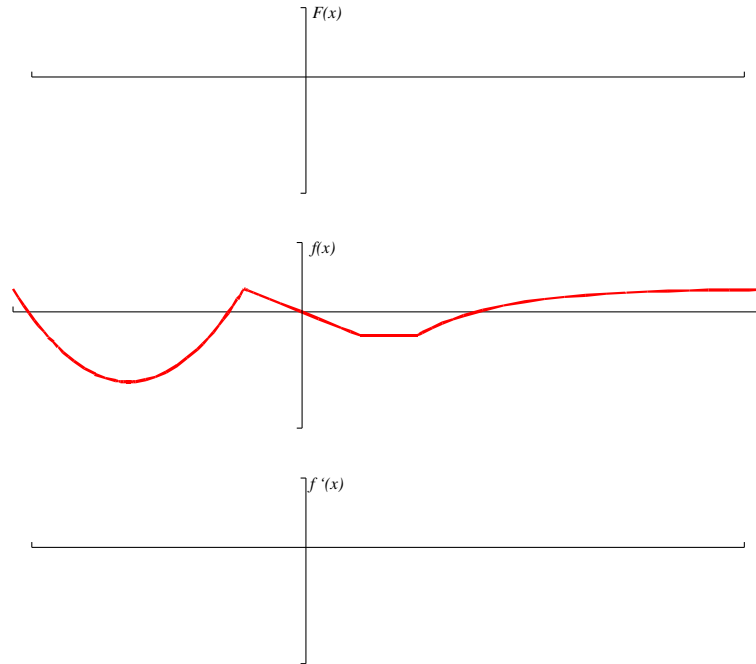
(d) Find $\frac{dy}{dr}$ if $y = \tan(e^{r^2} \arcsin(5r))$.

(e) Find $\frac{dy}{dx}$ if $y^3 + yx^2 + x^2 = 3y^2$.

(f) Find $\frac{dy}{dx}$ if $y = (1 + x^6)^{8x}$.

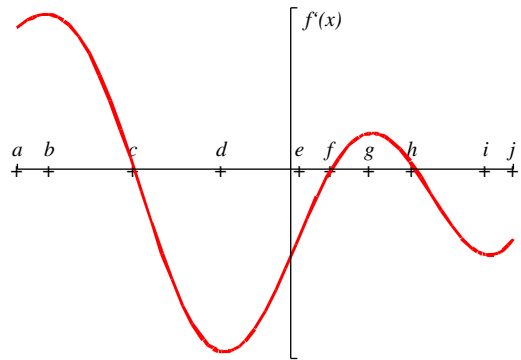
Hint: use logarithmic differentiation.

4. Given the graph of f , sketch a graph of f' and a graph of F , an antiderivative of f such that $F(0) = -1$.



5. Shown below is a graph of f' on its entire domain. The graph is NOT f .

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|--|-----------------------|
| At which x -value(s) | (b) f decreasing? |
| (a) does f have a stationary point? | (c) f' increasing? |
| (b) does f have a local max? | (d) f' decreasing? |
| (c) does f have a local min? | (e) f concave up? |
| (d) does f' have a stationary point? | (f) f concave down? |
| (e) does f' have a local max? | |
| (f) does f' have a local min? | |
| (g) is f greatest? | |
| (h) is f least? | |
| (i) is f' greatest? | |
| (j) is f' least? | |
| (k) is f'' greatest? | |
| (l) is f'' least? | |



- On what interval(s) is
- (a) f increasing?

6. Solve the IVP $y' = e^x - \sin x + 5$ given that $y(0) = 3$.

7. Evaluate the following limits.

(a) $\lim_{x \rightarrow \infty} \frac{x^2}{\ln x}$

(b) $\lim_{z \rightarrow 0} \frac{\sin(5z) - 5z}{z^3}$

(c) $\lim_{x \rightarrow 0} \frac{e^x - 1}{\cos x}$

(d) $\lim_{x \rightarrow 0^+} x^3 \ln x$

8. Consider the function $f(x) = x^6 - 2x^3$ on the interval $[-2, 2]$.

(a) Find the x - and y -coordinates of any and all critical points and classify each as a local maximum, local minimum, or neither.

(b) Find the x - and y -coordinates of any and all global extrema and classify each as a global maximum or global minimum.

(c) Find the x -coordinate(s) of any and all inflection points.