1. Let $A = \begin{bmatrix} 8 & -10 \\ 2 & -1 \end{bmatrix}$. Find the required matrices P and D that show why A is diagonalizable.

Hint: The vector $\mathbf{v} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ satisfies $A\mathbf{v} = 4\mathbf{v}$. We need to see if A has another eigenvalue...

so char poly $(A) = \begin{vmatrix} 8-\lambda & -10 \\ 2 & -1-\lambda \end{vmatrix} = (8-\lambda)(-1-\lambda) - (-10)(2)$

$= -8 + \lambda - 8\lambda + \lambda^2 + 20$

$= \lambda^2 - 7\lambda + 12 = (\lambda - 4)(\lambda - 3)$

we've been given an eigenvector corresponding to $\lambda = 4$ (namely $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$)

(but if you go on to find a basis for the eigenspace for $\lambda = 4$ you need solve $\begin{bmatrix} 8-4 & -10 \\ 2 & -1-4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$; $\begin{bmatrix} 4 & -10 \\ 2 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & -2.5 \\ 0 & 0 \end{bmatrix} \Rightarrow x_1 = 2.5x_2$ (x_2 free) $\rightarrow \begin{bmatrix} 2.5 \\ 1 \end{bmatrix}$ is an eig vector

note $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ is a multiple of $\begin{bmatrix} 2.5 \\ 1 \end{bmatrix}$; you can use either in P.)

for $\lambda = 3$, we have $\begin{bmatrix} 8-3 & -10 \\ 2 & -1-3 \end{bmatrix} = \begin{bmatrix} 5 & -10 \\ 2 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ where x_2 is free;

so $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is an eigenvector corresponding to $\lambda = 3$. One choice of D and P is then

$\begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$ & $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$. (there are other choices of course)

2. Let $\mathbf{a} = \begin{bmatrix} 5 \\ -2 \\ 7 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix} 11 \\ w \\ 5 \end{bmatrix}$.

2A. Suppose that H is a subspace of $\mathbb{R}^3$ and $\mathbf{a}$ is in H. Does $\mathbf{b}$ belong to $H^\perp$? Explain your answer.

Since $\vec{a} \in H$, in order for $\vec{b}$ to be in $H^\perp$ we'd need $\vec{a} \cdot \vec{b} = 0$.

But $\vec{a} \cdot \vec{b} = \begin{bmatrix} 5 \\ -2 \\ 7 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix} = 15 + 4 + 42 = 61 \neq 0$. So $\vec{a}$ & $\vec{b}$ are not orthogonal, so $\vec{b}$ cannot be in $H^\perp$.

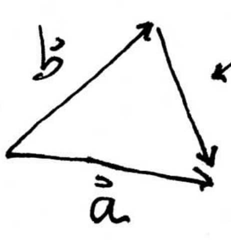
2B. Find w for which $\mathbf{c}$ and $\mathbf{a}$ are orthogonal.

We need $\vec{c} \cdot \vec{a} = 0$. But $\vec{c} \cdot \vec{a} = \begin{bmatrix} 11 \\ w \\ 5 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -2 \\ 7 \end{bmatrix} = 55 - 2w + 35 = 90 - 2w$

and $90 - 2w = 0 \Rightarrow w = 45$

2C. Find the distance from $\mathbf{b}$ to $\mathbf{a}$.

That's the length of this vector which is $\|\vec{a} - \vec{b}\|$ (or $\|\vec{b} - \vec{a}\|$)



$= \left\| \begin{bmatrix} 5 \\ -2 \\ 7 \end{bmatrix} - \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\| = \sqrt{2 \cdot 2 + 0 \cdot 0 + 1 \cdot 1}$

$= \sqrt{5}$