MATH 205A,B - LINEAR ALGEBRA
WINTER 2013

QUIZ 10

NAME: **Section:**(Circle one) A(1 : 10) B(2 : 40)

Show ALL your work CAREFULLY.

Let

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 0 & 2 \end{bmatrix}.$$

(a) Find an orthogonal basis for Col$A$.

Let $\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $\vec{u}_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Since the dot product $\vec{u}_1 \cdot \vec{u}_2 = 5$, the columns of A are NOT orthogonal. On the other hand, $\vec{u}_1$ is not a scalar multiple of $\vec{u}_2$ so these two vectors are linearly independent. Now, we use Gram-Schmidt Process to obtain an orthogonal basis for Col$A$**. First, let $\vec{v}_1 = \vec{u}_1$. Then, let**

$$\vec{v}_2 = \vec{u}_2 - \frac{\vec{u}_2 \cdot \vec{v}_1}{\vec{u}_2 \cdot \vec{v}_1}\vec{v}_1 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}.$$

It follows that

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \right\}$$

is an orthogonal basis for Col$A$**.**

(b) Find a QR decomposition for A, i.e., find a matrix Q and an upper triangular matrix R such that $A = QR$ and $Q^T Q = I$.

From the calculation in part (a), the vector $\vec{v}_1$ has length $\sqrt{5}$ and $\vec{v}_2$ has length 2. Thus, the matrix Q has columns $\vec{v}_1/\sqrt{5}, \vec{v}_2/2$ or

$$Q = \begin{bmatrix} 1/\sqrt{5} & 0 \\ 2/\sqrt{5} & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{so that} \qquad Q^T = \begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Now the upper triangular matrix is

$$R = Q^T A = \begin{bmatrix} \sqrt{5} & \sqrt{5} \\ 0 & 2 \end{bmatrix}.$$

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