

Name: _____

1. **Do not open this booklet until you are told to do so.**
 2. Try not to separate the pages. If they do become separated, write your names on every page and point this out to your proctor when you hand it in.
 3. Show an appropriate amount of work (including appropriate explanation) for each problem and not just the final answer. Include units in your answer where that is appropriate.
 4. You may use any calculator functionally equivalent to a TI-83/TI-83+ or TI-84/TI-84+. Use of calculators with more functionality than these is not allowed.
 5. **Turn off all cell phones and pagers, and remove all headphones.**
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Proficiency Level on Module 3:_____

Proficiency Level on Module 4:_____

Proficiency Level on Module 5:_____



Mathematician, physicist, and philosopher
- **Marie-Sophie Germain, born April 01, 1776**
A prime number p is a Sophie Germain prime if $2p + 1$ is also prime, eg. $\{2, 3, 5\}$

Module 3 - Problem 1

(I) The initial temperature of a hot cup of coffee served at Forage Market is 185° F and the café's temperature is kept at a constant 70° F. A student models the rate of change of the temperature difference between the cup of coffee and the café with time. The temperature of the cup of coffee at a time t after being served is $T(t)$. The student represents the temperature difference as y , i.e., $y = T(t) - 70$, and finds the differential equation

$$\frac{dy}{dt} = -ky.$$

If the temperature difference after 1 hour is 30° F, i.e. $y(1) = 30$, find the temperature of the cup of coffee after 30 minutes.

(II) Evaluate

$$\int \frac{e^5}{5x} + 4\ln 4 \cdot 4^x + 3\cos(7x) \, dx$$

(III) Let $f(x) = A\cos x + B\sin x$, where A and B are constants. Find values of A and B such that f is a solution of the IVP $f'' = -f$, $f'(\pi/3) = -1$, and $f(\pi/3) = \sqrt{3}$.

Module 3 - Problem 2

(I) Let $f(x) = \frac{x + \sin x}{\cos x}$. Find an equation of the line tangent to the graph of f at $x = 0$.

(II) Let $h(x) = f\left(\frac{g(x)}{x^2 + 1}\right)$. If $f(1) = 3$, $f(2) = 5$, $f'(1) = 7$, $f'(2) = 11$, $g(1) = 2$, and $g'(1) = 4$. Find $h'(1)$

Module 4 - Problem 1

(I) The equation $x^5 + xy^3 + x^2y + y^5 = 32$ implicitly defines a curve. What is the slope of this curve at the point $(0, 2)$.

(II) Rewrite

$$\sin(\arctan(2x))$$

as an algebraic expression. Be sure to draw the right angle triangle and label all the sides correctly.

Module 4 - Problem 2

(I) Show that

$$\frac{x}{(1+x^2)} \leq \arctan x \leq x,$$

for all $x \geq 0$ by first showing that the inequality holds for $x = 0$ and that a similar inequality holds for the derivatives of the functions.

(II) A wire of length 100 cm is to be cut into two pieces. One piece will be used to form a square; the other, to form a circle. How should the wire be cut to maximize the sum of the areas of the pieces?

Module 5 - Problem 1

(I) A spherical ball 8 inches in diameter is coated with a layer of ice of uniform thickness. If the ice melts at the rate of $10 \text{ in}^3/\text{min}$, how fast is the thickness of the ice decreasing when it is 2 inches thick? Recall that the volume of sphere of radius r is $\frac{4}{3}\pi r^3$.

(II) Find $\lim_{x \rightarrow \infty} \frac{4x^3}{8x^3 - 5x^2 + 10x + 100}$

(III) Suppose that $f(1) = 2$ and $f'(1) = 3$. Evaluate $\lim_{x \rightarrow 1} \frac{(f(x))^2 - 4}{x^2 - 1}$

(IV) Evaluate $\lim_{x \rightarrow 0^+} x^2 \ln x$

Module 5 - Problem 2

(I) Let $f(x) = x^7 - x^5 - x^4 + 2x + 1$.

- (a) Use the mean value theorem to explain why there must be a point c in the interval $(-1, 1)$ such that the line tangent to the graph of f at the point $(c, f(c))$ has slope 2.

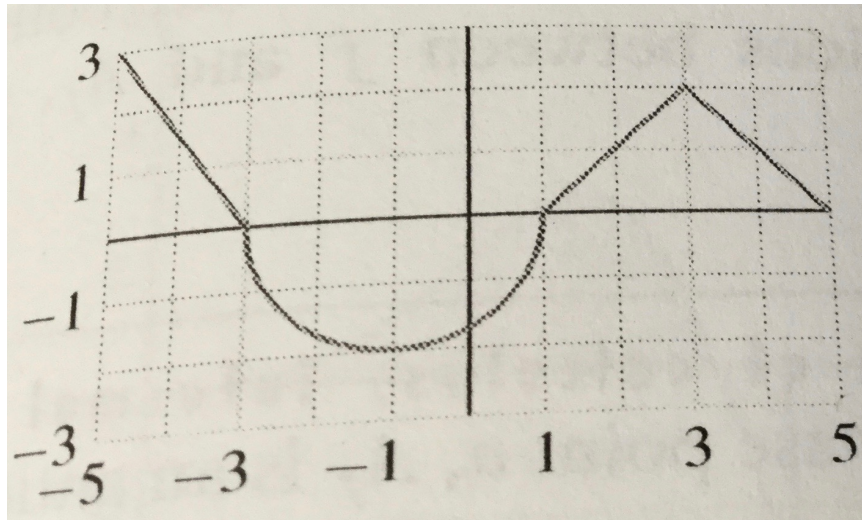
- (b) Find $f'(x)$. Use the intermediate value theorem to show that $f'(x) = 2$ at some point in the interval $[-1, 1]$.

(II) Given that $\int_0^4 f(x) dx = 16$, find the value of the integral

$$\int_{-4}^0 \left(\sqrt{16 - x^2} + f(x + 4) \right) dx$$

Module 5 - Problem 3

(I) The graph of f is shown below. [NOTE: The graph of f consists of straight lines and a semi-circle.]



(a) Evaluate $\int_{-5}^1 3f(x) dx$

(b) If $F(x) = \int_{-3}^x f(t) dt$,

(1) Find $F(3)$

(2) Where does F have a local minimum and why?