Name: Solutions

Math 105B: Winter 2013
Quiz 6: March 29

Correct answers accompanied by incorrect or incomplete work will not receive full credit. **Justify all answers.** Good Luck!

1. Use the Intermediate Value Theorem to show that $f(x)$ has a root between -1 and 2.

$$f(x) = (\sin x - 4)(x^2 + 3x - 2)$$

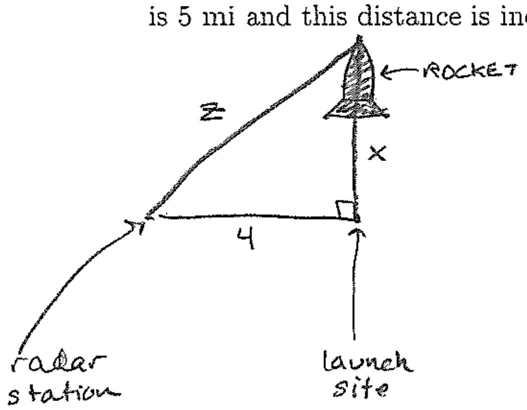
$\sin x - 4$ is continuous, so is x^2+3x-2. Thus their product is continuous, so $f(x)$ is continuous on $[-1, 2]$

$f(2) \approx -24.7$

$f(-1) \approx 19.4$

0 is between $f(2)$ and $f(-1)$. Thus the IVT says there is a "c" between -1 and 2 such that $f(c)=0$, i.e., there is a root between -1 and 2.

2. A rocket that is launched vertically is tracked by a radar station located on the ground 4 mi from the launch site. What is the vertical speed of the rocket at the instant its distance from the radar station is 5 mi and this distance is increasing at the rate of 3600 mi/h?



we know $\frac{dz}{dt}$, want $\frac{dx}{dt}$

$$x^2 + 16 = z^2$$

$$2x\frac{dx}{dt} = 2z\frac{dz}{dt} \quad (*)$$

plug known values into $(*)$

$(z=5 \Rightarrow x^2+16 = 25 \Rightarrow x=3)$

$$2(3)\frac{dx}{dt} = 2(5)3600$$

$$\frac{dx}{dt} = \frac{(5)3600}{3} = \boxed{6000 \text{ mi/h}}$$

3. Evaluate the following limit. If you use L'Hopital's rule verify that you can use it.

$$\lim_{x\to\infty} \frac{(\ln x)^2}{x} = \frac{\lim_{x\to\infty} (\ln x)^2}{\lim_{x\to\infty} x} = \frac{\infty}{\infty}$$

so we can us L'H

$$\lim_{x\to\infty} \frac{(\ln x)^2}{x} \overset{L'H}{=} \lim_{x\to\infty} \frac{2(\ln x)\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{2\ln x}{x} = \frac{\lim_{x\to\infty} 2\ln x}{\lim_{x\to\infty} x} = \frac{\infty}{\infty}$$

so we can use L'H again

$$\lim_{x\to\infty} \frac{2\ln x}{x} \overset{L'H}{=} \lim_{x\to\infty} \frac{1/x}{1} = \boxed{0}$$