**Math 205 A & B** Quiz 07 ver01 page 1 03/28/2014 Name: Suggested sol'n

**1.** Let $A = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 4 & 0 \\ 6 & 3 & 7 \end{bmatrix}$; let $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$.

1A. Find $A\mathbf{v}$. The result tells you one eigenvalue of $A$. What is that eigenvalue?

$$A\vec{v} = \begin{bmatrix} 2+0+3 \\ 0+0+0 \\ 6+0-21 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -15 \end{bmatrix} = 5\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} \therefore \boxed{\lambda = 5}$$

1B. FACT: Another eigenvalue of $A$ is $\lambda = 4$ and it has multiplicity 2 (you do NOT have to show this). Find a basis for the eigenspace corresponding to $\lambda = 4$.

we need the null space of $A - 4I = \begin{bmatrix} -2 & -1 & -1 \\ 0 & 0 & 0 \\ 6 & 3 & 3 \end{bmatrix}$. The RREF is $\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

telling us the sol'ns of $(A-4I)\vec{v} = \vec{0}$ are

$\vec{v} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2\begin{bmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} -\frac{1}{2} \\ 0 \\ 1 \end{bmatrix}$ where $x_2$ & $x_3$ are free; a basis for the eigenspace corresponding to $\lambda = 4$ is $\left\{\begin{bmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} \\ 0 \\ 1 \end{bmatrix}\right\}$

1C. Find a pair of matrices $P$ and $D$ that show $A$ is diagonalizable (according to our definition of diagonalization); just give $P$ and $D$ and label them (you do NOT need to find $P^{-1}$ or compute $PDP^{-1}$). If $A$ is not diagonalizable, explain why it isn't.

from 1A an eigen value is $\lambda = 5$; from 1B an eigenvalue is $\lambda = 4$. Since dim(eigspace cor. to $\lambda = 4$) + dim(eig space cor. to $\lambda = 5$) must be $\leq 3$ and dim(eig space for $\lambda = 4$) is 2, it follows that dim(eig. space cor. to $\lambda = 5$) is 1; $\therefore$ from 1A we have $\left\{\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}\right\}$ is a basis of that eig space.

Therefore $A$ is diagonalizable, with

$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$

(there are many variations possible.

e.g. $D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, $P = \begin{bmatrix} -\frac{1}{2} & \frac{1}{3} & -\frac{1}{2} \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

**2.** Let $\mathbf{a} = \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 4 \\ x \\ 4 \end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix}$.

2A) Find a unit vector in the direction of $\mathbf{a}$.

such a vector is $\frac{1}{\|\vec{a}\|}\vec{a} = \frac{1}{\sqrt{9+16+25}}\begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix} = \boxed{\frac{1}{\sqrt{50}}\begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}}$ or $\begin{bmatrix} \frac{3}{5\sqrt{2}} \\ -4/5\sqrt{2} \\ 5/5\sqrt{2} \end{bmatrix}$ ...

2B) Find all values of $x$ for which $\mathbf{a} \perp \mathbf{b}$.

we need $\vec{a} \cdot \vec{b} = 0$; $12 - 4x + 20 = 0$;

$32 = 4x$; $\boxed{x = 8}$

2C) Find the distance between $\mathbf{a}$ and $\mathbf{c}$; show your work.

By definition, that distance is $\|\vec{a} - \vec{c}\|$

$$= \left\|\begin{bmatrix} -7 \\ -9 \\ 4 \end{bmatrix}\right\| = \sqrt{49 + 81 + 16}$$

$$= \boxed{\sqrt{146}} \ (\approx 12.08\ldots)$$