

1. Let  $A = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 4 & 0 \\ 6 & 3 & 7 \end{bmatrix}$ ; let  $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$ .

1A. Find  $A\mathbf{v}$ . The result tells you one eigenvalue of  $A$ . What is that eigenvalue?

1B. FACT: Another eigenvalue of  $A$  is  $\lambda = 4$  and it has multiplicity 2 (you do NOT have to show this). Find a basis for the eigenspace corresponding to  $\lambda = 4$ .

1C. Find a pair of matrices  $P$  and  $D$  that show  $A$  is diagonalizable (according to our definition of diagonalization); just give  $P$  and  $D$  and label them (you do NOT need to find  $P^{-1}$  or compute  $PDP^{-1}$ ). If  $A$  is not diagonalizable, explain why it isn't.

2. Let  $\mathbf{a} = \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}$ ,  $\mathbf{b} = \begin{bmatrix} 4 \\ x \\ 4 \end{bmatrix}$  and  $\mathbf{c} = \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix}$ .

2A) Find a unit vector in the direction of  $\mathbf{a}$ .

2B) Find all values of  $x$  for which  $\mathbf{a} \perp \mathbf{b}$ .

2C) Find the distance between  $\mathbf{a}$  and  $\mathbf{c}$ ; show your work.