Math 205 A & B Quiz 07 ver01 page 1 03/27/2015 **Name** suggested solution

1. Let $A = \begin{bmatrix} 6 & 1 & 0 \\ 12 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. **1A.** Find the characteristic polynomial of A in factored form. Show all your work.

That polynomial is $\det(A - \lambda I) = \begin{vmatrix} 6-\lambda & 1 & 0 \\ 12 & 5-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 6-\lambda & 1 \\ 12 & 5-\lambda \end{vmatrix}$

$$= (2-\lambda)\big((6-\lambda)(5-\lambda) - 12\big)$$
$$= (2-\lambda)(30 - 11\lambda + \lambda^2 - 12)$$
$$= (2-\lambda)(\lambda^2 - 11\lambda + 18)$$
$$= (2-\lambda)(\lambda - 9)(\lambda - 2)$$
$$= (-1)(\lambda-2)(\lambda-9)(\lambda-2) \quad \text{or} \quad (-1)(\lambda-2)^2(\lambda-9)$$

1B. Use (1A) to list all the eigenvalues of A along with their respective multiplicities:

so: $\lambda = 2$ with multiplicity 2 and $\lambda = 9$ with multiplicity 1.

2. Let $B = \begin{bmatrix} 6 & -4 & 2 \\ 1 & 2 & 1 \\ 1 & -2 & 5 \end{bmatrix}$. It's a **fact** (you don't need to show this) that $\lambda = 4$ is an eigenvalue of B of multiplicity 2.

2A. Find a basis of the eigenspace corresponding to $\lambda = 4$. Show all your work, starting by explicitly finding $B - 4I_3$.

This is the same question as "find a basis of the null space of $B - 4I$"

Now, $B - 4I = \begin{bmatrix} 2 & -4 & 2 \\ 1 & -2 & 1 \\ 1 & -2 & 1 \end{bmatrix}$ has rref $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, so the solutions of $(B-4I)\vec{x} = \vec{0}$ are given by

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

where x_2 & x_3 are free. A basis is then: $\left\{\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right\}$

2B. Find $B\mathbf{c}$, where $\mathbf{c} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$.

use a calculator to get $\begin{bmatrix} 12 \\ 16 \\ 20 \end{bmatrix}$

2C. Show that $\mathbf{c} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ is in the eigenspace of $\lambda = 4$ by writing $\mathbf{c}$ as a linear combination of the basis vector(s) you found in (2A). (Give the LC explicitly).

That easy-to-use basis gives $4\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + 5\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$

(although you should "verify" the top row comes out right: $4 \cdot 2 + 5 \cdot -1 \stackrel{?}{=} 3$? yes indeed.)

2D. Compute $B\,\mathbf{k}$ for $\mathbf{k} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$. What do you discover from this?

$B\vec{k}$ is $\begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}$ which is $5\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ showing $\lambda = 5$ is another eigenvalue of B.

2E. Using the discovery in (2D) and the **fact** given at the start of problem **(2)**, find the characteristic polynomial of B in factored form (no need to multiply it all out).

Since $\lambda = 4$ has mult. 2 we know $(\lambda - 4)^2$ is one factor.
Since the degree of this poly is 3 and $\lambda = 5$ is also an eigen vector, $(\lambda - 5)$ must be the other factor
so char poly B is $(\lambda - 4)^2(\lambda - 5)$