MATH 205A,B - LINEAR ALGEBRA
WINTER 2013

QUIZ 9

NAME: **Section:**(Circle one) A(1 : 10) B(2 : 40)

Show ALL your work CAREFULLY.

(a) Let

$$\vec{u}_1 = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 3 \\ -6 \\ 5 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}.$$

Determine whether the set $S = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ is an orthogonal set. Justify your answer.

The set S is orthogonal if and only if each of the following dot products $\vec{u}_1 \cdot \vec{u}_2$, $\vec{u}_2 \cdot \vec{u}_3$, $\vec{u}_3 \cdot \vec{u}_1$ is zero. Since $\vec{u}_1 \cdot \vec{u}_2 = (-1)(3) + (2)(-6) + (3)(5) = 0$, $\vec{u}_2 \cdot \vec{u}_3 = (3)(2) + (-6)(1) + (5)(0) = 0$, and $\vec{u}_3 \cdot \vec{u}_1 = (2)(-1) + (1)(2) + (0)(3) = 0$, we conclude that S is an orthogonal set.

(b) Let $\vec{y} = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}$ and $\vec{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$. Find the projection of $\vec{y}$ onto $W = \text{Span}\{\vec{u}\}$.

The projection of $\vec{y}$ onto W is given by

$$\begin{aligned} \text{proj}_W \vec{y} &= \frac{\vec{y} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} \vec{u} \\ &= \frac{(4)(1) + (-2)(-1) + (3)(2)}{1^2 + (-1)^2 + 2^2} \vec{u} \\ &= \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}. \end{aligned}$$

(c) Let $\vec{y}$ and W be as in part (b). Find the shortest distance between $\vec{y}$ and the line (through origin) W.

The closest point in W to $\vec{y}$ is $\text{proj}_W \vec{y}$ so the shortest distance in question is the length

$$\|\vec{y} - \text{proj}_W \vec{y}\| = \left\| \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix} \right\| = \sqrt{5}.$$

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