1. Let $\mathbf{b}_1 = \begin{bmatrix} 3 \\ 9 \\ 7 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{p}_1 = \begin{bmatrix} 6 \\ 27 \\ 26 \end{bmatrix}$, and $\mathbf{p}_2 = \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix}$. Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and let $\mathcal{P} = \{\mathbf{p}_1, \mathbf{p}_2\}$.

It's a fact that $\mathcal{B}$ and $\mathcal{P}$ are bases for the same subspace H of $\mathbb{R}^3$; you do not need to prove this.

1A: Find $[\mathbf{b}_1]_{\mathcal{P}}$.

1B: Find the change-of-basis matrix M from $\mathcal{B}$ to $\mathcal{P}$.

2. Let $K = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 3 & 3 & 8 & 0 \\ 4 & 6 & 7 & 0 \\ 0 & 0 & 5 & 2 \end{bmatrix}$.

Make good use of the 0's in the matrix K to find the determinant of K using the "cofactor expansion" method we've used in class. Show all the intermediate steps and results. (You can check your final answer on your calculator, of course).

3. Let $S = \begin{bmatrix} 1 & -3 \\ 4 & 8 \end{bmatrix}$. Find the characteristic polynomial of S, and the eigenvalues of S.