

Math 105 Quiz 7

§4.8, 4.9, 4.5

Name: Key

Show all work for credit. Make sure to mention any assumptions made to use conclusions of theorems.

1. At 2:00PM a car passed the 20 mile marker on the highway. At 2:10PM the car passed the 35 mile marker on the highway. Explain how and why you know the car travelled 90 mph at some point during the trip.

The MVT holds because position is a continuous function and differentiable on $[2:00\text{PM}, 2:10\text{PM}]$. MVT says that there must be a time between 2:00 and 2:10 when the car was travelling the average rate which is $\frac{f(2:10) - f(2:00)}{10\text{mins} = 1\text{hr}} = 90\text{mph}$.

2. Could the above car have been traveling 100mph during the trip?

Yes, it could have, but we don't know.

3. Use the Intermediate Value Theorem to show that there is a root of $f(x) = \cos(x) - x$ in the interval $[0, \pi]$.

$$f(0) = 1$$

$$f(\pi) = -1 - \pi$$

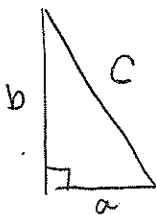
Since $f(x)$ is continuous

on $[0, \pi]$, the IVT says

that any value between 1 and $-1 - \pi$ is obtained by some x . 0 is between

1 and $-1 - \pi$, so there exists a root x where $f(x) = 0$.

4. A right triangle has sides a and b and a hypotenuse c . The side a starts at length 5 and is increasing at a rate of 3ft per second. The side b starts at length 12ft and is decreasing at a rate of 2 ft per second. After 3 seconds, at what rate is the hypotenuse changing?



$$a^2 + b^2 = c^2$$

$$2a \frac{da}{dt} + 2b \frac{db}{dt} = 2c \frac{dc}{dt}$$

$$2(14)(3) + 2(6)(-2) = 2(15.23) \frac{dc}{dt}$$

$$14^2 + 6^2 = c^2$$

$$c \approx 15.23$$

$$60 = 30.46 \frac{dc}{dt}$$

$$\frac{dc}{dt} = \boxed{1.972 \text{ ft/sec}}$$