

Name: _____

Exam 2 – Math 105

Show all your work to receive full credit for a problem. There are a total of 72 points on this test. Good luck!

1. (4 points each.) Suppose f and g are differentiable functions with the values for the function and derivative given in the following table:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	7	9	5
3	1	9	7	9
5	9	5	5	3
7	7	1	3	1
9	5	3	1	7

(a) If $h(x) = f(x) + 2g(x)$, find $h'(1)$.

(b) If $h(x) = \frac{xf(x)}{g(x)}$, find $h'(5)$.

(c) If $h(x) = f(g(x^2))$, find $h'(3)$.

2. (4 points each.) Evaluate the following limits.

(a) $\lim_{x \rightarrow 0} \frac{1 - \cos(3x)}{8x^2}$

(b) $\lim_{x \rightarrow 0} \frac{x3^x}{3^x - 1}$

(c) $\lim_{x \rightarrow \infty} \left(3 + \frac{10}{x^2} \right)$

3. (7 points each.) Each of the following derivatives contains one or more mistakes. Find the mistakes, explain what was done wrong, and compute the correct derivative.

(a) $f(x) = x \arctan(4x)$, $f'(x) = \frac{4x}{1 + 4x^2}$

(b) $f(x) = \ln(x \ln x)$, $f'(x) = \frac{2 \ln x}{x}$

4. (6 points.) Solve the initial value problem $y'(t) = 8e^t + 1$ where $y(0) = 5$.

5. (8 points.) Use logarithmic differentiation to find the derivative of the function

$$f(x) = (\cos x)^x.$$

6. (8 points.) Find $\frac{dy}{dx}$ when y is implicitly given by the equation

$$xe^y = y \sin x$$

7. (12 points) A landscape architect plans to enclose a 3000 square foot rectangular region in a botanical garden. She will use shrubs costing \$25 per foot along three sides and fencing costing \$10 per foot along the fourth side. Find the minimum total cost.

8. (2 points EXTRA CREDIT.) Below is a unit circle. Use it to find the following exact values:

- (a) $\arccos(-1)$
- (b) $\arctan(1/\sqrt{3})$

