Calculus I - Section A **Module $\{2, 3, 4\}$ Proficiency Assessment** **March 18, 2016**

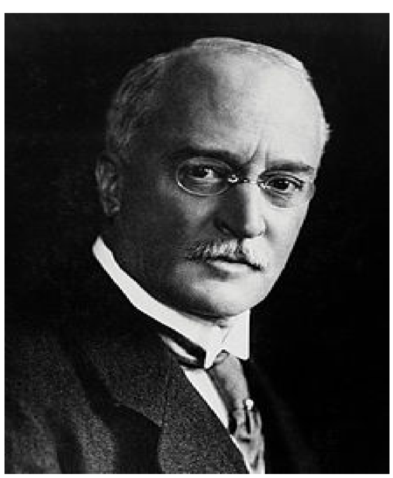
Name: ____

1. **Do not open this booklet until you are told to do so.**
2. Try not to separate the pages. If they do become separated, write your names on every page and point this out to your proctor when you hand it in.
3. Show an appropriate amount of work (including appropriate explanation) for each problem and not just the final answer. Include units in your answer where that is appropriate.
4. You may use any calculator functionally equivalent to a TI-83/TI-83+ or TI-84/TI-84+. Use of calculators with more functionality than these is not allowed.
5. **Turn off all cell phones and pagers, and remove all headphones.**

Proficiency Level on Module 2:____________

Proficiency Level on Module 3:____________

Proficiency Level on Module 4:____________



Inventor of the diesel engine

- Rudolf Diesel, born March 18, 1858

Module 2 - Problem 1

(**I**)

(a) Evaluate the limit $\lim_{x\to 4}\left(\frac{2x}{x+4}+\frac{x-4}{x^2-16}\right)$

(b) Evaluate $\lim_{h\to 0}\frac{f(4+h)-f(4)}{h}$, where $f(x)=3x^2+4\sqrt{x}+5$

(c) Evaluate $\lim_{\theta\to 0}\frac{\sin(5\theta)}{7\theta}$

(**II**) Find

$$\int 6x^{11}-\frac{1}{x^3}-2x+\frac{\pi}{x^{3/2}}\ dx$$

Module 2 - Problem 2

(**I**) Use calculus to prove that $f(x) = 3x^4 - 8x^3 + 17 > 0$ for every x.

(**II**) Find values of a and b so that the line $2x + 3y = a$ is tangent to the graph of $f(x) = bx^2$ at $x = 3$.

(**III**) What is the maximum perimeter of the rectangle whose base is on the x-axis and whose two upper corners are on the curve $y = 2(1 - x^2)$

Module 3 - Problem 1

(**I**) The number of rabbits in Central Park in New York City grows at a rate proportional to the number present. If there were 100 rabbits in Central park in 1986 and 120 rabbits in 1987. How many rabbits were in Central Park in 2006 if miraculously none of the rabbits had died since 1986?

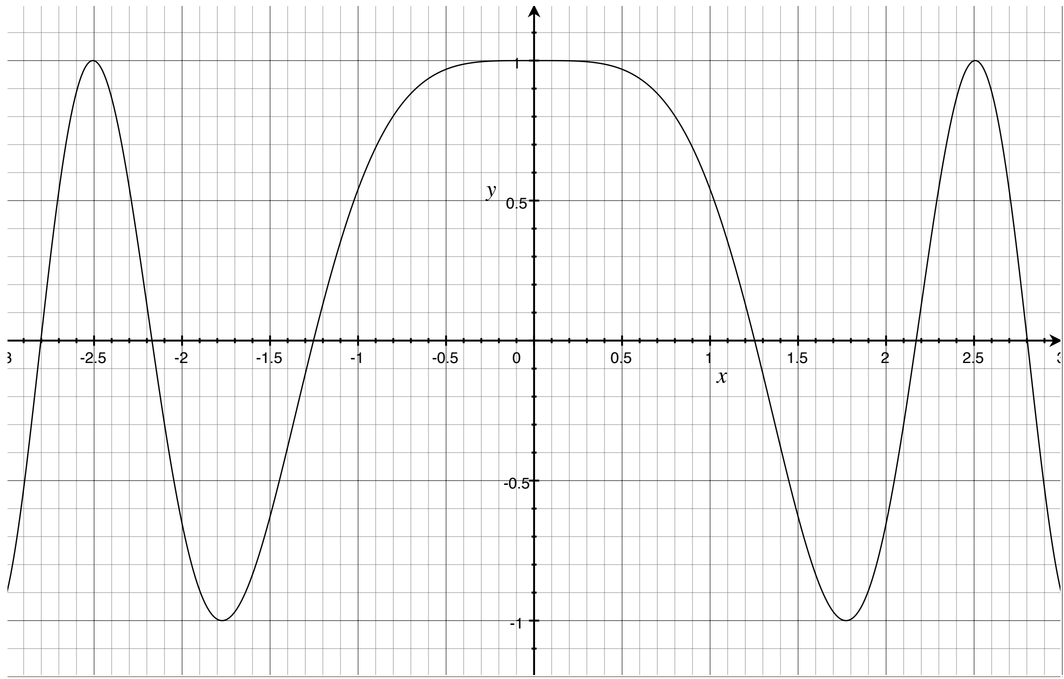
(**II**) Evaluate

$$\int \frac{5}{x} + \ln 5 \cdot 5^x + 3\sin(4x)\ dx$$

(**III**) Let $f(x) = 2 - \sin x$ and $g(x) = 1 + \cos x$. Find the maximum vertical distance between the curves $y = f(x)$ and $y = g(x)$ over the interval $[0, 2\pi]$.

Module 3 - Problem 2

(**I**) The graph of the function f is shown below.



$$\text{Let} \quad h(x) = \sin\left(f(x)\right), \quad m(x) = \ln[x \cdot f(x)], \quad p(x) = \frac{2^x f(x)}{e^x + \tan x}.$$

(a) Show that h has a stationary point at $x = 2.5$

(b) What is the concavity of h at $x = 2.5$?

(c) Is $m(x)$ increasing or decreasing at $x = -1.5$?

(d) Find $p'(0)$.

Module 4 - Problem 1

(**I**) Find the equation of the line tangent to the curve $y^3 = x^2 - 8\cos(x \cdot y)$ at the point $(0, -2)$

(**II**) Rewrite

$$\cos(\arctan(2x))$$

as an algebraic expression. Be sure to draw the right angle triangle and label all the sides correctly.

Module 4 - Problem 2

(**I**) Find $f'(x)$ if $f(x) = x^2 \arctan(\sqrt{x})$.

(**II**) Find

$$\int \frac{e^x}{\sqrt{1 - e^{2x}}}\, dx$$

(**II**) An open top box with a volume of $\frac{125}{6}$ cubic feet is to be twice as long as it is wide. The material for the box costs \$0.10 per square foot. What are the dimensions of the least expensive box? How much does it cost?