

Math 105: Review for Exam II

1. Find dy/dx for each of the following.

(a) $y = x^2 + 2^x + e^2 + e^{2x} + \ln 2 + \ln(2x) + (\ln 2)x + \arctan 2$

(b) $y = \sqrt{x} \cdot \arctan(5x)$

(c) $y = \ln(\tan(2^{\cos(x^2)}))$

(d) $y = \frac{x + e^\pi}{\cos 4 + \sin^5(6x)}$

2. Consider the curve defined by $x^3 + y^3 = \frac{9}{2}xy$ (known as the Folium of Descartes).

(a) Find dy/dx .

(b) Verify that the point $(1,2)$ is on the curve above.

(c) Find the equation of the tangent line at the point $(1,2)$.

3. Evaluate the following limits.

(a) $\lim_{x \rightarrow 1} \frac{x^3 - 1}{7 - 7x}$

(b) $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{3^x}$

(c) $\lim_{x \rightarrow 0} \frac{1 - \cos(4x)}{5x^2}$

(d) $\lim_{x \rightarrow \infty} \frac{x^2}{2^x}$

4. Consider the function $f(x) = x^4 e^x$ with domain all real numbers.

(a) Find the x -value(s) of all roots (x -intercepts) of f .

(b) Find the x - and y -value(s) of all critical points and identify each as a local max, local min, or neither.

(c) Find the x - and y -value(s) of all global extrema and identify each as a global max or global min.

(d) Find the x -value(s) of all inflection points.

(e) Sketch f .

5. How would your answers to the previous question change if the domain of f were $[-10, 10]$?
6. (Please omit this problem.) Use the Intermediate Value Theorem to explain why $f(x) = x^3 - 4x^2 + 5$ must have a root somewhere on the interval $[1, 2]$.
7. Find an antiderivative of $y = \frac{5}{\sqrt{1-9x^2}} + x^3 + \cos(2x) + e^3$.
8. You are planning to build a box-shaped aquarium with no top and with two square ends. Your budget is \$288. If the glass for the sides costs \$12 per square foot and the opaque material for the bottom costs \$3 per square foot, what dimensions will maximize the volume? Be sure to show how you know you have found the maximum.