

Name: \_\_\_\_\_

YOUR GRADE IS BASED ON CORRECTNESS, COMPLETENESS, AND CLARITY ON EACH EXERCISE. EXPLAIN ALL ANSWERS COMPLETELY. YOU MAY USE A CALCULATOR, BUT NO NOTES, BOOKS, OR OTHER STUDENTS. GOOD LUCK!

1.) (10 pts.) Use the given table to compute and evaluate the derivatives of the functions below.

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	7	9	5
9	5	3	1	7

a.) Compute the derivative of  $y = xf(x)$ , then evaluate when  $x = 1$ .

b.) Compute the derivative of  $y = f(g(x))$ , then evaluate when  $x = 1$ .

2.) (15 pts.) Compute the derivative of  $y = \frac{x^2 + 4x + 3}{x + 3}$  two ways. Then compare your answers.

a.) (5 pts.) Do not simplify first. Use the Quotient Rule. Simplify AFTER computing the derivative.

b.) (5 pts.) Simplify the fraction FIRST. Then use the Power Rule to compute the derivative.

c.) (5 pts.) Compare your two fully simplified final answers. Are they the same? *Should* they be?

3.) (15 pts.)

a.) (5 pts.) Use implicit differentiation to compute the derivative of  $x^2y + y^3 = 75$ .

b.) (5 pts.) Solve for  $\frac{dy}{dx}$ .

c.) (5 pts.) Determine the slope of the tangent line to the curve in part (a) at the point  $(x, y) = (4, 3)$ .

4.) (15 pts.)

a.) (5 pts.) Use a reference triangle to write  $\cos(\arctan(5x))$  as an algebraic expression, that is, one not involving any trigonometric functions. (Note: you are not being asked to compute a derivative here.)

b.) (5 pts.) Compute the derivative of  $\ln(\arcsin(x^2))$ .

c.) (5 pts.) Compute an antiderivative of  $g(x) = \frac{e^x + \sec^2 x + 2x}{e^x + \tan x + x^2 + \pi}$ .

5.) (15 pts.)

a.) (5 pts.) Sketch a graph (it is fine to use your calculator) of  $y = \frac{40x^4 + 4x^2 - 1}{10x^4 + 8x^2 + 1}$  and use your graph to make a guess about  $\lim_{x \rightarrow \infty} \frac{40x^4 + 4x^2 - 1}{10x^4 + 8x^2 + 1}$ . (Be sure to show *how* you are using your graph to make your guess.)

b.) (5 pts.) Use an algebraic or calculus-based technique to confirm your guess from part (a).

c.) (5 pts.) Use L'Hôpital's Rule to compute  $\lim_{x \rightarrow 0} \frac{e^x - 1}{x^2 + 3x}$ . Be sure to confirm that this is a case in which you *can* use L'Hôpital's Rule.

6.) (15 pts.) Suppose an airline policy states that all baggage must be box-shaped with a sum of length, width, and height not exceeding 64 inches. What are the dimensions and volume of a square-based box with the greatest volume under these conditions? (Note: use calculus to solve this and to confirm that your result gives *maximum* volume.)

- 7.) (15 pts.) Consider the function  $y = \frac{1}{2}x + \sin x$  on the interval  $[0, 2\pi]$ .
- a.) (5 pts.) At which  $x$ -values, from 0 to  $2\pi$ , does  $y$  have stationary points? Use calculus to produce your answer, and give the  $x$ -values *exactly*, not as decimal approximations.
- b.) (5 pts.) Demonstrate, using the First Derivative Test and/or the Second Derivative Test, whether each of your stationary points from part (a) is a local maximum or a local minimum.
- c.) (5 pts.) How many points of inflection does  $y$  have for  $x$ -values in the interval  $[0, 2\pi]$ ? Use calculus to support your answer, and as in part (a), give all  $x$ -values *exactly*.

**BONUS:** The mathematical definition of *continuity at a point* “ $a$ ” involves the concept of a limit. Explain this connection. (You can state the definition of continuity at a point, or you can give a more intuitively-worded explanation that involves limits.) You may use the back of this page if you would like more space to write.