TEST 2 Solutions

Math 105
3/16/12

Name: _____
by writing my name I swear this work is my own

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 6 problems and is worth 100 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (*14 points*)

x	$f(x)$	$g(x)$	$j(x)$	$f'(x)$	$g'(x)$	$j'(x)$
-2	0	1	-1	3	2	1
-1	1	3	2	-1	3	-2
0	2	1	1	2	-2	2
1	3	1	-1	0	3	1
2	-2	2	1	3	0	3
3	-1	1	-1	1	-2	0

a. (*7 pts*) $H(x) = f(j(x)) + 2g(x)$. Find $H'(2)$.

$$H'(x) = f'(j(x)) \cdot j'(x) + 2g'(x)$$
$$H'(2) = f'(j(2)) \cdot j'(2) + 2g'(2) = 0$$

b. (*7 pts*) $F(x) = \dfrac{xj(x)}{f(x)^2}$. Find $F'(0)$.

$$F'(x) = \frac{(xj'(x) + j(x))f(x)^2 - 2f(x)f'(x)xj(x)}{f(x)^4}$$
$$F'(0) = \frac{(1)(2)^2 - 2(2)(2)(0)(1)}{2^4} = 1/4$$

2. (*21 points*) Find y' in **3** of **4** of the following. If you do more than four, then clearly mark which three you want graded. If you don't, the worst three will be chosen for you.

1. $y = \dfrac{\sin^4(x)\tan^2(x)}{(x^2+1)^2}$ using logarithmic differentiation

$$y' = \left(\frac{4\cos x}{\sin x} + \frac{2\sec^2 x}{\tan x} - \frac{4x}{x^2+1}\right)\left(\frac{\sin^4 x \tan^2 x}{(x^2+1)^2}\right)$$

2. $y = \log_6(x^2e^x)$

$$y' = \frac{2xe^x + x^2e^x}{\ln(6)x^2e^x}$$

3. $y = \arcsin(x^2-1) + 4\pi^2 + \sqrt[3]{x^2}$

$$y' = \frac{2x}{\sqrt{1-(x^2-1)^2}} + \frac{2}{3\sqrt[3]{x}}$$

4. $y = \dfrac{\arctan(2x)}{x^2+1} + e^3$

$$y' = \frac{\frac{2}{1+4x^2}(x^2+1) - (2x)(\arctan 2x)}{(x^2+1)^2}$$

3. (*12 points*)

a. (*8 pts*) For the equation $e^{xy} = (x-2y)^2$ use implicit differentiation to find $\frac{dy}{dx}$.

$$e^{xy}(y + xy') = 2(x-2y)(1-2y') = 2x - 4xy' - 4y + 8yy'$$

$$y' = \frac{2x - 4y - ye^{xy}}{xe^{xy} + 4x - 8y}$$

b. (*4 pts*) Determine $\frac{dy}{dx}$ at the point (1,0).

$$\frac{2}{5}$$

4. (*12 points*) Find an antiderivative of the given function using an educated guess and check.

a. (*6 pts*) $f(x) = \dfrac{4}{9+x^2}$.

$$F(x) = \frac{4}{3}\arctan\left(\frac{x}{3}\right) + C$$

Check:

$$F'(x) = \frac{1}{1+(x/3)^2} * 1/3 * 4/3 = \frac{4}{9+x^2}$$

b. (*6 pts*) $g(x) = \dfrac{2\sin x \cos x}{1+\sin^2 x}$.

$$G(x) = \ln(1+\sin^2 x) + C$$

Check:

$$G'(x) = \frac{1}{1+\sin^2 x} \cdot 2\sin x \cos x$$

5. (*20 points*) Evaluate the following limits. Only use L'Hôpital's rule when appropriate. Show your work!!

a. (*5 pts*) $\lim_{x\to\infty} \frac{1}{x}(\ln x)^2$

$$\lim_{x\to\infty} \frac{1}{x}(\ln x)^2 = \frac{(\ln x)^2}{x}(TYPE\frac{\infty}{\infty})$$
$$= \lim_{x\to\infty} \frac{2\ln x}{x}(TYPE\frac{\infty}{\infty})$$
$$= \lim_{x\to\infty} \frac{2}{x} = 0$$

b. (*5 pts*) $\lim_{x\to 0} \frac{4x^2+2x+1}{3x^2+1}$

$$1$$

c. (*5 pts*) $\lim_{x\to\infty} \frac{4x^2+2x+1}{3x^2+1}$

$$\lim_{x\to\infty} \frac{4x^2+2x+1}{3x^2+1}(TYPE\frac{\infty}{\infty})$$
$$= \lim_{x\to\infty} \frac{8x+2}{6x}(TYPE\frac{\infty}{\infty})$$
$$= \lim_{x\to\infty} \frac{8}{6} = \frac{4}{3}$$

d. (*5 pts*) $\lim_{x\to\infty} x^{\frac{1}{\ln(x)}}$

$$y = \lim_{x\to\infty} x^{\frac{1}{\ln(x)}}$$
$$\ln y = \lim_{x\to\infty} \frac{\ln(x)}{\ln(x)} = 1$$
$$y = e$$

6. (*21 points*) Speeders Beware!
It has been found that for every 5mph you travel over 55mph, you decrease your gas mileage by 7%. You are renting a car for a 400 mile trip. The car rental costs $15/hour. Gas is $4.25/gallon.
When traveling up to 55mph the gas mileage is 27miles/gallon. After 55mph, the car's gas mileage drops by 7% for each 5mph over 55mph.

a. (*2 pts*) If you travel at a constant speed of 55mph, how many hours will you travel to complete the trip?

$$\frac{400}{55} = 7.27 hours$$

b. (*3 pts*) How much would the trip cost if you travelled 55mph the entire trip?

$$Cost = \$15 * (7.27) + \frac{400}{27} * \$4.25 = \$172.01$$

c. (*7 pts*) You would like to minimize the cost on a trip with constant speed. Write the function for cost. Let t be the number of hours and x be the speed.

$$Cost = 15t + 4.25\left(\frac{400}{27(1-.07)^{\frac{x-55}{5}}}\right)$$

d. (*3 pts*) What is the constraint?

$$400 = xt$$

e. (*3 pts*) Write the objective function in terms of a single variable and simplify as much as possible.
Two options:

$$Cost = 15t + 4.25\left(\frac{400}{27(1-.07)^{\frac{\frac{400}{t}-55}{5}}}\right)$$

$$Cost = 15\frac{400}{x} + 4.25\left(\frac{400}{27(1-.07)^{\frac{x-55}{5}}}\right)$$

f. (*3 pts*) Describe, DO NOT CALCULATE, how you would finish the problem.
I would find the first derivative of the cost function and determine when it is 0. Then I would check these critical points using the second derivative to determine whether the function has a minimum or a maximum at those critical values. I want the second derivative to be positive which will imply a minimum. I would then determine the speed.