

Math 105 — Second Midterm

March 15, 2013

Name: _____

Instructor: _____ Section: _____

1. **Do not open this exam until you are told to do so.**
2. This exam has 9 pages including this cover AND IS DOUBLE SIDED. There are 8 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions.
5. Show an appropriate amount of work (including appropriate explanation). Include units in your answer where that is appropriate. Time is of course a consideration, but do not provide no work except when specified.
6. You may use any previously permitted calculator. However, you must state when you use it.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones and hats.
9. Remember that this is a chance to show what you've learned, and that the questions are just prompts.

Problem	Points	Score
1	15	
2	18	
3	10	
4	13	
5	14	
6	14	
7	14	
8	02	
Total	100	

1. [15 points] Suppose that $W(h)$ is an invertible function which tells us how many gallons of water an oak tree of height h feet uses on a hot summer day.

a. [6 points] Give a practical (one sentence) interpretation for each of the following quantities or statements.

• $W(50) = 25$.

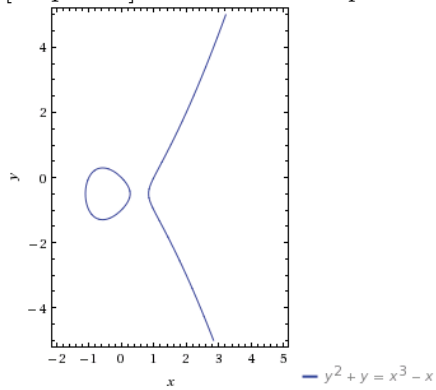
• $W^{-1}(40)$

• $\frac{d}{dx}[W^{-1}](40) = 2$

b. [9 points] Use the following table to calculate $\frac{d}{dx}[W^{-1}](40)$, and $W^{-1}(30)$ and $W'(50)$.

h	10	20	30	40	50	60
$W(h)$	5	15	30	35	40	70
$W'(h)$	2	1	2	4	1	5

2. [18 points] There is an *elliptic curve* given by the equation $y^2 + y = x^3 - x$.



- a. [2 points] Find any point that is NOT on the curve. Prove that it is not on the curve.

- b. [2 points] Check that the point $(2, 2)$ is on the given elliptic curve.

- c. [8 points] Find the slope of the tangent line at $(2, 2)$.
- d. [6 points] Use the previous parts to approximate the value of x on the curve when $y = 2.5$.

3. [10 points] Answer the following problems True/False/Neither. Answer TRUE if it MUST BE true; answer FALSE if it MUST BE false; answer NEITHER otherwise. No explanations necessary, no partial credit. Your answer must be clear...if I cannot tell what your intended answer is, then it is wrong.

a. [2 points] If $\tan(x) = 3$ then $\arctan(x) = 1/3$.

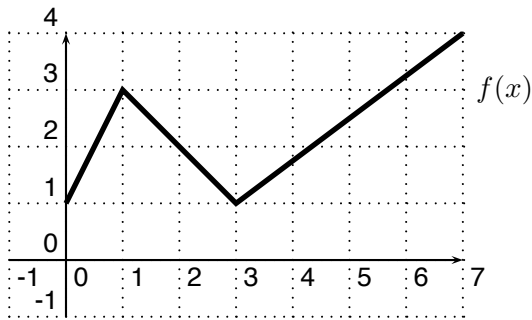
b. [2 points] $\frac{d}{dx}[x^x] = xx^{x-1}$

c. [2 points] A continuous function always has a local maximum.

d. [2 points] The derivative of 2^x is $x2^{x-1}$.

e. [2 points] A continuous function from the interval $[-2, 3]$ always has a global maximum.

4. [13 points]



x	1	2	3	4	5	6
$g(x)$	0	4	0	-18	-56	-120
$g'(x)$	6	1	-10	-27	-50	-79
$g''(x)$	-2	-8	-14	-20	-26	-32

a. [6 points] Let $h(x) = \frac{g(x)}{f(2x+3)}$. Find $h'(1)$ or explain why it doesn't exist.

b. [7 points] Let $k(x) = g(g(x))$. Determine whether $k(x)$ is increasing or decreasing at $x = 2$.

5. [14 points]

Let $f(x) = e^{ax^2+b}$, where a and b are constants.

a. [7 points] Find the critical points of $f(x)$. Your answer should depend on a and b .

b. [7 points] Find the inflection points of $f(x)$. Your answer should depend on a and b .

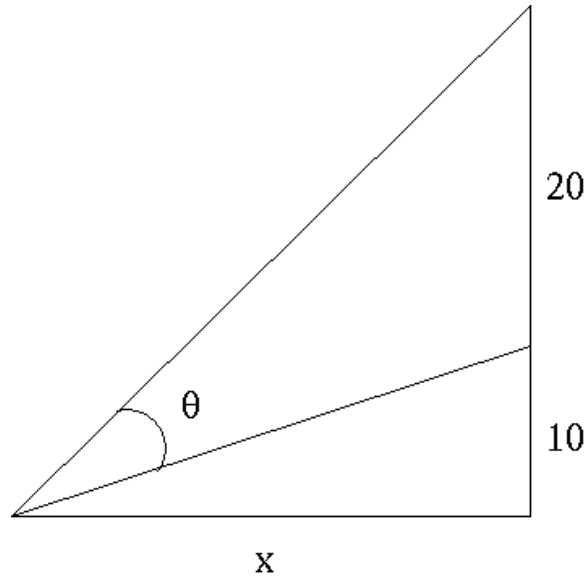
6. [14 points]

Consider all right triangles formed by the positive x and y axes and a line through the point $(4, 3)$. Which such triangle minimizes the length of the hypotenuse? (Hint: Draw a picture! Draw several...then label them with variables.)

7. [14 points]

A movie screen is 10 feet off the ground and is an additional 20 feet tall. How far from the screen should you seat yourself so that your viewing angle, θ , is as large as possible? (Hint:

The derivative of $\arctan(x) = \frac{1}{1+x^2}$)



8. [2 points] What is your favorite comedic movie?