Name:____

Math 105: Winter 2013
Exam 2: March 15

Correct answers accompanied by incorrect or incomplete work will not receive full credit.

1. (20 points) Let $f(x) = \arcsin\sqrt{x}$.

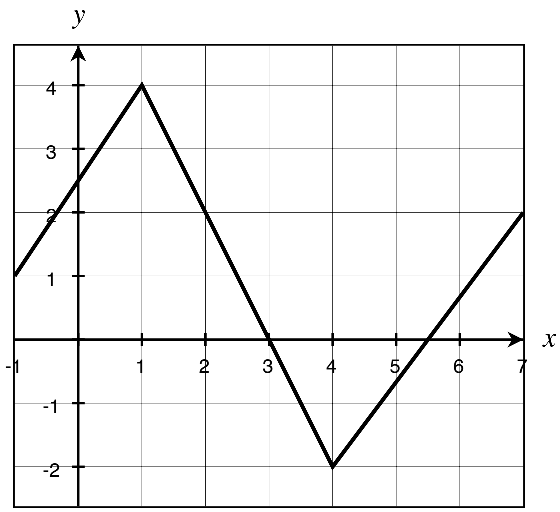
   (a) Find $f\left(\frac{3}{4}\right)$. Express your answer exactly, *not* as a decimal approximation.

   (b) Find $f'(x)$.

   (c) Find $f'\left(\frac{3}{4}\right)$. Express your answer as a decimal approximation.

   (d) Write the equation of the line tangent to $f(x)$ at $x = \frac{3}{4}$.

2. (18 points) Consider a function g that has the following graph.



Let $h(x) = \dfrac{x^{4/3}}{g(x)}$.

(a) Write an expression for $h'(x)$.

(b) Is h increasing at $x = 2$? Justify your answer using calculus.

(c) For what values of x in the interval $[-1, 7]$ is $h'(x)$ undefined?

3. (16 points) Find y' in **2 of 3** of the following. If you do more than two, then clearly mark which two you want graded. If you don't, the **worst** two will be chosen for you.

   (a) $y = 5e^{(x^3+2x)} + \sin^2 x + \cos\frac{\pi}{7}$

   (b) $y = 3^x \arcsin x + \ln(\cos x) + e^7$

   (c) $y = \arctan\left(x^3 + \cos\left(x^2 + 8x\right)\right)$

4. (8 points) Find $\frac{dy}{dx}$ in **1 of 2** of the following. If you do more than one, then clearly mark which one you want graded. If you don't, the **worst** will be chosen for you.

   (a) $y = (\cos x)^x$

   (b) $\cos y + xy^2 = 15y$

5. (8 points) Find the solution to the Initial Value Problem

$$y' = \frac{9}{x} + 2^x + 6x^2 + 17, \quad y(1) = 5.$$

6. (12 points) Let f be a function with first and second derivatives given below:

$$f'(x) = \frac{\sin x - \frac{1}{2}}{x-2}, \qquad f''(x) = \frac{(x-2)\cos x - (\sin x - \frac{1}{2})}{(x-2)^2}$$

Find the *critical points* of f on the interval $[0, 2\pi]$. Give the x-values exactly, *not* as decimal approximations. Classify each critical point as a local maximum, local minimum, or neither.

7. (18 points) An outdoor track is to be created in the shape of a rectangle with semicircles attached on two opposite ends of the rectangle. The track must have a perimeter of 440 yards. Find the dimensions for the track that maximize the area of the *rectangular* (shaded) portion of the field enclosed by the track. (Helpful formulas related to circles: Area $= \pi r^2$, Circumference $= 2\pi r$.)

(a) What is the objective function for the quantity you are trying to optimize?

(b) What is the constraint equation?

(c) Answer the question. (Be sure you use calculus to verify that your critical point optimizes the objective function.)