

TEST 2

Math 105
3/14/14

Name: _____

by writing my name I swear this work is my own

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 8 problems and is worth 100 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!
- Some useful formulas:

Area of a circle with radius r , $A = \pi r^2$,

Circumference of a circle with radius r , $C = 2\pi r$.

Area of a rectangle with sides x and y , $A = xy$,

Perimeter of a rectangle with sides x , y , $P = 2x + 2y$

Volume of a rectangular prism with sides x, y, z , $V = xyz$,

Surface area of rectangular prism with x, y, z , $SA = 2xy + 2yz + 2xz$

Volume of a cylinder with radius r and height h , $V = \pi r^2 h$,

Surface area of a cylinder with radius r and height h , $SA = 2\pi r^2 + 2\pi r h$

1. (14 points)

x	$f(x)$	$g(x)$	$j(x)$	$f'(x)$	$g'(x)$	$j'(x)$
-2	0	1	-1	3	2	1
-1	1	3	2	-1	3	0
0	2	1	1	2	-2	2
1	3	1	-1	1	3	1
2	-2	2	1	3	0	3
3	-1	1	-1	1	-2	2

a. (7 pts) $H(x) = f(g(x)) + \frac{j(x)}{x+1}$. Find $H'(2)$.

b. (7 pts) $F(x) = g(x)^3 - x^2 f(x)$. Find $F'(1)$.

2. (8 points) Find $f'(x)$ using logarithmic differentiation.

$$f(x) = \frac{\cos^4(x)(10x^3 - 2x)^3}{e^{x^2} \sin(2x)}$$

3. (15 points) Find $f'(x)$.

a. (9 pts) $f(x) = \sqrt[3]{(\cos^2(x) + 3 + \sin^2(x))} + \arcsin\left(\frac{2}{x}\right) + xe^{\tan(x)}$

b. (6 pts) $f(x) = 2^{3x} \cos(4x) + \ln(\sin(2x))$

4. (16 points)

a. (8 pts) For the equation $e^{\cos(y)} = x^4 \arctan(y)$ use implicit differentiation to find $\frac{dy}{dx}$.

b. (8 pts) Let a be a constant, find the equation of the tangent line at $(a, 0)$ of

$$x^{2/3} + y^{2/3} = a^{2/3}$$

5. (15 points) Find the antiderivative of the given function.

a. (5 pts) $f(x) = \frac{1}{\sqrt{1-4x^2}}.$

b. (5 pts) $g(x) = \frac{\arctan(x)}{1+x^2}.$

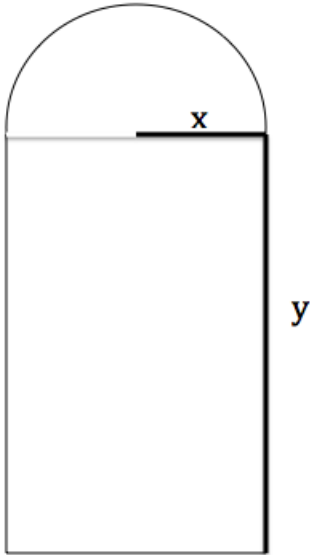
c. (5 pts) $h(x) = \frac{1}{(\arctan(x))(1+x^2)}.$

6. (14 points) Evaluate the following limits. Only use L'Hôpital's rule when appropriate. Show your work!!

a. (6 pts) $\lim_{x \rightarrow 0} \frac{x - \sin(x)}{x^2}$

b. (8 pts) $\lim_{x \rightarrow \infty} x \frac{\ln(k)}{\ln(x) + 1},$ for k a constant.

7. (14 points) Consider the following window formed by a rectangle and a semi-circle. There is 50 feet of metal piping to go around the perimeter of the window. What are the dimensions (x and y) to maximize the area of the window?



8. (4 points) For the following optimization problem, draw a picture, find a function for the objective function, and find a function for the constraint function. DO NOT SOLVE.

A kaleidoscope is a cylinder with a glass top, glass base, and cardboard all around. If the cardboard costs \$2 per sq. ft and the glass costs \$5 per sq.ft, then what dimensions (radius and height) maximize the volume of the kaleidoscope if we only have \$20 to spend?

9. (2 points) BONUS: Who discovered L'Hôpital's rule?