# MATH 205A,B - LINEAR ALGEBRA
# WINTER 2013

QUIZ 8

**NAME:** **Section:**(Circle one) A(1 : 10) B(2 : 40)

**Show ALL your work CAREFULLY.**

Let

$$A = \begin{bmatrix} 3 & 0 & 0 \\ -2 & 5 & 3 \\ 8 & 1 & 3 \end{bmatrix}.$$

(a) The matrix $A$ has 3 as one of its eigenvalues. Find the other eigenvalues of $A$.

**The eigenvales of $A$ are the solutions to the characteristic equation $\det(A - \lambda I) = 0$. It follows that**

$$\begin{aligned} \det(A - \lambda I) &= \det \begin{bmatrix} 3-\lambda & 0 & 0 \\ -2 & 5-\lambda & 3 \\ 8 & 1 & 3-\lambda \end{bmatrix} \\ &= (3-\lambda)[(5-\lambda)(3-\lambda) - (1)(3)] \\ &= (3-\lambda)[\lambda^2 - 8\lambda + 12] \\ &= (3-\lambda)((\lambda - 6)(\lambda - 2). \end{aligned}$$

**We now conclude that the other eigenvalues of $A$ are 2 and 6.**

(b) Find a basis for the eigenspace of $A$ corresponding to the eigenvalue $\lambda = 3$.

**When $\lambda = 3$, the matrix**

$$A - 3I = \begin{bmatrix} 0 & 0 & 0 \\ -2 & 2 & 3 \\ 8 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 8 & 1 & 0 \\ -2 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1/8 & 0 \\ -2 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1/8 & 0 \\ 0 & 9/4 & 3 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1/8 & 0 \\ 0 & 1 & 4/3 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1/6 \\ 0 & 1 & 4/3 \\ 0 & 0 & 0 \end{bmatrix}$$

**Thus the eigenspace of $A$ corresponding to $\lambda = 3$ is**

$$\{x_3 \begin{bmatrix} 1/6 \\ -4/3 \\ 1 \end{bmatrix} : x_3 \text{ in } \mathbb{R}^2\} \quad \textbf{with basis } \{\begin{bmatrix} 1/6 \\ -4/3 \\ 1 \end{bmatrix}\}.$$

(c) Determine the rank of the matrix $(A - 3I)$. [Hint: what is the dimension of the eignespace as in part (b)?] Justify your answer.

**The rank of $(A - 3I)$ is the dimension of the column space of $(A - 3I)$. The eigenspace in part (b), which is one dimensional, is the same as the null space of $(A - 3I)$. Since $(A - 3I)$ is a $3 \times 3$ matrix, by the rank theorem, the rank of $(A - 3I)$ is equal to $3 - \dim \mathrm{Nul}(A - 3I) = 3 - 1 = 2$.**

*Date*: March 13, 2013.