Math 105: Review for Exam II

1. Find dy/dx for each of the following.

   (a) $y = x^2 + 2^x + e^2 + e^{2x} + \ln 2 + \ln(2x) + (\ln 2)x + \arctan 2$

   (b) $y = \sqrt{x} \cdot \arctan(5x)$

   (c) $y = \ln(\tan(2^{\cos(x^2)}))$

   (d) $y = \dfrac{x + e^{\pi}}{\cos 4 + \sin^5(6x)}$

   (e) $y = (x^2 + 1)^{\sin x}$

2. Consider the curve defined by $x^3 + y^3 = \dfrac{9}{2}xy$ (known as the Folium of Descartes).

   (a) Find dy/dx.

(b) Verify that the point (1,2) is on the curve above.

(c) Find the equation of the tangent line at the point (1,2).

3. Evaluate the following limits. [Students in the 8:00 and 1:10 sections may omit this problem.]

(a) $\lim_{x \to 1} \frac{x^3 - 1}{7 - 7x}$

(b) $\lim_{x \to 0} \frac{1 - \cos(2x)}{3^x}$

(c) $\lim_{x \to 0} \frac{1 - \cos(4x)}{5x^2}$

(d) $\lim_{x \to \infty} \frac{x^2}{2^x}$

4. Rewrite $\tan(\arccos x)$ as an algebraic expression - no trigonometric or inverse trigonometric functions. [Students in the 1:10 section may omit this problem.]

5. Consider the function $f(x) = x^4 e^x$ with domain all real numbers.

(a) Find the x-value(s) of all roots (x-intercepts) of f.

(b) Find the x- and y-value(s) of all critical points and identify each as a local max, local min, or neither.

(c) Find the x- and y-value(s) of all global extrema and identify each as a global max or global min.

(d) Find the x-value(s) of all inflection points.

(e) Sketch f.

6. How would your answers to the previous question change if the domain of f were $[-10, 10]$?

7. Use the Intermediate Value Theorem to explain why $f(x) = x^3 - 4x^2 + 5$ must have a root somewhere on the interval $[1, 2]$. [Students in the 8:00 and 1:10 sections may omit this problem.]

8. You are planning to build a box-shaped aquarium with no top and with two square ends. Your budget is \$288. If the glass for the sides costs \$12 per square foot and the opaque material for the bottom costs \$3 per square foot, what dimensions will maximize the volume? Be sure to show how you know you have found the maximum.