

Mid-term Exam #2
MATH 106 - A&B Winter 2016

Name: _____

Instructions:

- Answer as many of the following questions as possible.
- No cell phones or collaboration allowed. If you leave the classroom during the exam you must leave your cell phone with the instructor.
- Approved calculators are allowed.
- Additional scrap paper is available upon request.
- *Multiple choice questions:* Circle the letter corresponding to your answer. No partial credit will be awarded.
- *Short answer questions:* Show all of your work on the page of the problem. Clearly indicate your answer and the reasoning that you used to arrive at the answer. You do not have to simplify algebraic expressions.

This exam has 4 multiple choice problems and 5 short answer problems. There are a total of 100 points.

Good luck!

Problem	Possible Points	Points Earned
MC	20	
5	30	
6	16	
7	20	
8	8	
9	6	
TOTAL	100	

1. (5 points) Which of the following is the correct form for the partial fraction decomposition of

$$\frac{x+5}{x^2(x^2+1)}?$$

- A. $\frac{A}{x} + \frac{Bx+C}{x^2+1}$
- B. $\frac{A}{x^2} + \frac{B}{x^2+1}$
- C. $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$
- D. $\frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$
- E. $\frac{Ax+B}{x^2} + \frac{Cx+D}{x^2+1}$

2. (5 points) Which substitution is needed to evaluate the integral

$$\int \tan^4 x \sec^2 x \, dx?$$

- A. $u = \cos x, du = -\sin x \, dx$
- B. $u = \sec x, du = \tan x \sec x \, dx$
- C. $u = 1 - \tan^2 x, du = 2 \tan x \sec^2 x \, dx$
- D. $u = \tan x, du = \sec^2 x \, dx$
- E. $u = \tan^4 x, du = \sec^2 x \, dx$

3. (5 points) Suppose that the third-order Maclaurin polynomial for a function f is given by

$$P_3(x) = 4 + \frac{x}{6} - \frac{2x^2}{3} - 12x^3.$$

What is $f''(0)$?

- A. $f''(0) = -\frac{4}{3}$
- B. $f''(0) = -\frac{2}{3}$
- C. $f''(0) = \frac{1}{6}$
- D. $f''(0) = \frac{1}{2}$
- E. $f''(0) = 0$

4. (5 points) The improper integral $\int_{-\infty}^{\infty} x dx$ is

- A. convergent since the area to the left of $x = 0$ cancels with the area to the right of $x = 0$.
- B. convergent since it equals $\lim_{t \rightarrow -\infty} \int_t^0 x dx + \lim_{t \rightarrow \infty} \int_0^t x dx = -\infty + \infty = 0$.
- C. divergent by comparison to $\int_{-\infty}^{\infty} xe^{-x} dx$
- D. divergent since $\int_{-\infty}^0 x dx$ is divergent and $\int_0^{\infty} x dx$ is convergent.
- E. divergent since both integrals $\int_{-\infty}^0 x dx$ and $\int_0^{\infty} x dx$ are divergent.

5. (30 points) Evaluate the following integrals using calculus. Show your work.

(a) (10 points) $\int \frac{5x + 8}{x^2 + 2x - 8} dx$

(b) (10 points) $\int \frac{\sqrt{x^2 - 1}}{x} dx$

(c) (10 points) Compute ONE of the following indefinite integrals. *You will not receive extra credit for attempting both problems. If you attempt both problems make it clear which problem that you would like graded.*

i. $\int \arcsin(x) dx$

ii. $\int \sin(\ln x) dx$

6. (16 points) Determine if the following integrals are convergent or divergent. If convergent, evaluate the integral using calculus. Show your work.

(a) (8 points) $\int_1^9 \frac{1}{(x-4)^2} dx$

(b) (8 points) $\int_{-\infty}^{-2} \frac{dx}{1-x}$

7. (20 points) For this problem, consider $f(x) = \ln(x^2)$.

(a) (10 points) Find the fourth-order Taylor polynomial $P_4(x)$ for $f(x)$ centered at $x_0 = 1$. *You do not have to simplify coefficients.*

(b) (10 points) Taylor's Theorem states that $|f(x) - P_n(x)| \leq \frac{K_{n+1}}{(n+1)!} |x - x_0|^{n+1}$ for all values of x in an interval I containing x_0 . What is the maximum error committed by using $P_4(x)$ (as in part (a)) over the interval $[\frac{1}{2}, \frac{3}{2}]$? *You do not have to simplify your answer.*

8. (8 points) Use comparison to determine if the following improper integral converges or diverges. If the integral converges, find an upper bound for its value. Show all of your work to justify your answer.

$$\int_1^{\infty} \frac{x^2 + 1}{x^4 + x} dx$$

9. (6 points) Suppose that the probability density function of a random variable X is as follows:

$$f(x) = \begin{cases} ce^{-x/3} & \text{for } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) (4 points) Find the value of the constant c .

- (b) (2 points) Find the probability that $X \leq \frac{1}{4}$.