1. Use integration by parts to find $\int 3x^2 \arctan(x)\, dx$. Show all your work. (Hint: this problem may involve division of polynomials later on).

2. Show how the substitution $x = 2\cos t$ can be used to find $\int x^3 \left(\sqrt{4-x^2}\right)^5 \, dx$. (Same as $\int x^3 (4-x^2)^{5/2} \, dx$). Show all your work and any relevant triangles.

3A. Find $P_2(x)$, the second-order Taylor polynomial approximation of $f(x) = \sqrt[3]{x-12}$, in powers of $x - 20$. Show all your work neatly organized in a table as we've done in class. Constants in your polynomial must be written as fractions (not their decimal equivalents)

3B. You can use P_2 to find an approximation of $\sqrt[3]{6}$ by evaluating $P_2(18)$, since $P_2(18) \approx f(18) = \sqrt[3]{18-12} = \sqrt[3]{6}$. Find the maximum possible error in using $P_2(18)$ to approximate $f(18)$ as guaranteed by Taylor's theorem. In your work, choose K_3 correct to 4 decimal places.

3C. Find $P_2(18)$ and also the exact error if $P_2(18)$ is indeed used to approximate $f(18)$.

4. Use the method of partial fractions to find $\displaystyle\int \frac{4\,x^2 + 27\,x + 71}{(x-1)\,(x^2 + 8\,x + 25)}\,dx$

5. Explain why $\displaystyle\int_2^5 \frac{2x}{\sqrt{x^2-4}}\,dx$ is an improper integral. Then determine if it diverges or converges, and if it does converge, find out to what. Use good notation throughout.

6A. Let $f(x) = 4\,x\,e^{-2x}$. Explain why f is a probability density function on $[0, \infty)$. You can use the fact that an anti-derivative of $f(x)$ is $-(1 + 2x)e^{-2x}$. Support any claims about limits with a brief table.

6B. Suppose $f(x)$ represents the distribution of how many *decades* LED lightbulbs last before failure. What integral represents the probability that a lightbulb chosen at random will last between 10 and 15 years? (That is, between 1 and 1.5 decades)? Find this probability; show your work.

6C. What is the probability that a bulb will last more than 15 years?