*** * * NOTE! this quiz is TWO SIDED! * * ***

1. Consider the following vectors in $\mathbb{P}_4$: Let $\mathbf{v}_1 = 3x^4 + 5x^2 + 6$, $\mathbf{v}_2 = x^4 + 2x^2 + 3$ and $\mathbf{v}_3 = 5x^4 + 7x^2 + 4$.
Let $\mathbf{b}$ be the polynomial $49x^4 + 73x^2 + 58$.

1A) In terms of the unknowns α_1, α_2 and α_3, what system of equations do you need to set up to determine if $\mathbf{b}$ can be written as a linear combination $\alpha_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3 = \mathbf{b}$?

We seek $\alpha_1, \alpha_2, \alpha_3$ for which

$$\alpha_1(3x^4+5x^2+6) + \alpha_2(x^4+2x^2+3) + \alpha_3(5x^4+7x^2+4) = 49x^4+73x^2+58$$

Equating coefficients of corresponding powers of x, we find

$$\begin{cases} 3\alpha_1 + \alpha_2 + 5\alpha_3 = 49 \\ 5\alpha_1 + 2\alpha_2 + 7\alpha_3 = 73 \\ 6\alpha_1 + 3\alpha_2 + 4\alpha_3 = 58 \end{cases}$$

((the 3 eqn's result from comparing the x^4, x^2 and constant terms, respectively. Since no x^3 or x terms appear, there's no need to write any additional equations such as " $0\alpha_1 + 0\alpha_2 + 0\alpha_3 = 0$ (from the x^3 term) but there's nothing wrong with it, either))

1B) Now determine the values of α_1, α_2 and α_3 or explain why there are none. Show any RREF'd matrices you use.

The rref of the augmented matrix corresponding to the system in 1A is

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 7 \end{array}\right],$$ telling us $\alpha_1 = 4$ $\alpha_2 = 2$ $\alpha_3 = 7$

1C) Without setting up any equations or finding any RREF's, give a quick reason why $\mathbf{c} = 5x^3 + 7x^2 + 11$ is obviously not a linear combination of of $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$.

The coefficient of x^3 in each of $\vec{v}_1, \vec{v}_2, \vec{v}_3$ is 0, and so therefore 0 is also the coefficient of x^3 in any Linear Combination of them. Since the coefficient of x^3 in $\vec{c}$ is 5, not 0, $\vec{c}$ is not a linear combination of $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

((the answer has NOTHING to do with the lack of an x^4-term in $\vec{c}$. Also, some students incorrectly said that $\vec{c}$ is not in $\mathbb{P}_4$. But it IS because $\deg(\vec{c})$ is ≤ 4.))

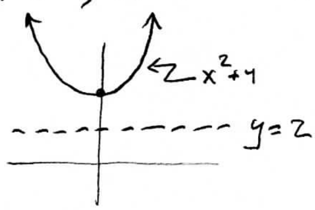
*** * * * This quiz CONTINUES on the OTHER SIDE * * * ***

2. Let H be the set of functions in $\mathbf{F}$ whose graphs are completely above the horizontal line $y = 2$.

2A. Is $\mathbf{v}_1 = x^2 + 4$ in H? You can draw a graph of $\mathbf{v}_1$ (copy it here) to explain your answer.

yes $\vec{v}_1$ is in H. Since $x^2 \geq 0$, $x^2+4 \geq 4 > 2$ for all x's.

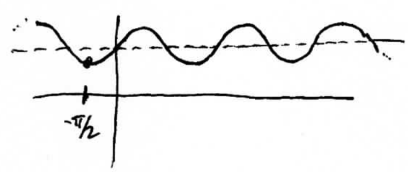
The graph supports this:



2B. Is $\mathbf{v}_2 = 2 + \sin x$ in H? You can draw a graph of $\mathbf{v}_2$ (copy it here) to explain your answer.

Absolutely NOT. for example, if $x = -\pi/2$, then $2 + \sin(-\pi/2) = 2 - 1 = 1$ and so at $x = -\pi/2$ the graph of $\vec{v}_2$ is <u>NOT</u> above the line $y = 2$.

The graph shows this clearly:



2C. Explain informally why H is closed under the vector addition of $\mathbf{F}$ (hint: think about "addition of y-coordinates") or give an explicit counter example.

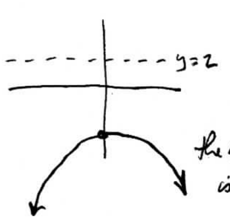
If two vectors $\vec{v}_1$ and $\vec{v}_2$ belong to H, then their graphs are completely above $y = 2$. Therefore the function represented by $\vec{v}_1 + \vec{v}_2$ has y coordinates above $y = 4$ (since addition of two #'s each greater than 2 yields a sum greater than 4) so certainly the graph is above $y = 2$ since $4 > 2$.

Thus if $\vec{v}_1$ and $\vec{v}_2$ are in H, so is their sum, and $\therefore$ H <u>IS</u> closed under vector addition of $\mathbb{F}$.

2D. Explain informally why H is closed under scalar multiplication (hint: think about "multiplying y-coordinates by any arbitrary scalar)" or give an explicit counter example that shows H is not closed under scalar multiplication.

Negative scalars cause a huge problem here! for example, consider $\vec{v}_1 = x^2 + 4$, and $r = -1$. While $\vec{v}_1$ <u>IS</u> in H, the graph of $r\vec{v}_1$ is completely below $y = -2$, so certainly not above $y = 2$. That is, $r\vec{v}_1 \notin H$. so it's a counter example.

H is <u>NOT</u> closed under scalar multiplication



the graph of $-1\vec{v}_1$ is <u>NOT</u> above $y=2$