

**\* \* \* NOTE! this quiz is TWO SIDED! \* \* \***

1. Consider the following vectors in  $\mathbb{P}_4$ : Let  $\mathbf{v}_1 = 3x^4 + 5x^2 + 6$ ,  $\mathbf{v}_2 = x^4 + 2x^2 + 3$  and  $\mathbf{v}_3 = 5x^4 + 7x^2 + 4$ .

Let  $\mathbf{b}$  be the polynomial  $49x^4 + 73x^2 + 58$ .

1A) In terms of the unknowns  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$ , what system of equations do you need to set up to determine if  $\mathbf{b}$  can be written as a linear combination  $\alpha_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3 = \mathbf{b}$ ?

1B) Now determine the values of  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  or explain why there are none. Show any RREF'd matrices you use.

1C) Without setting up any equations or finding any RREF's, give a quick reason why  $\mathbf{c} = 5x^3 + 7x^2 + 11$  is obviously not a linear combination of  $\mathbf{v}_1$ ,  $\mathbf{v}_2$  and  $\mathbf{v}_3$ .

**\* \* \* \* This quiz CONTINUES on the OTHER SIDE \* \* \* \***

2. Let  $H$  be the set of functions in  $\mathbf{F}$  whose graphs are completely above the horizontal line  $y = 2$ .

2A. Is  $\mathbf{v}_1 = x^2 + 4$  in  $H$ ? You can draw a graph of  $\mathbf{v}_1$  (copy it here) to explain your answer.

2B. Is  $\mathbf{v}_2 = 2 + \sin x$  in  $H$ ? You can draw a graph of  $\mathbf{v}_2$  (copy it here) to explain your answer.

2C. Explain informally why  $H$  is closed under the vector addition of  $\mathbf{F}$  (hint: think about “addition of  $y$ -coordinates”) or give an explicit counter example.

2D. Explain informally why  $H$  is closed under scalar multiplication (hint: think about “multiplying  $y$ -coordinates by any arbitrary scalar”) or give an explicit counter example that shows  $H$  is not closed under scalar multiplication.