

**MATH 205A,B - LINEAR ALGEBRA
WINTER 2013**

QUIZ 7

NAME:

Section:(Circle one) A(1 : 10) B(2 : 40)

Show **ALL** your work **CAREFULLY**.

(a) Find the coordinate vector $[\vec{x}]_{\mathbf{B}}$ relative to the basis $\mathbf{B} = \{\vec{b}_1, \vec{b}_2\}$ where

$$\vec{b}_1 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}, \vec{b}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \text{ and } \vec{x} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

The coordinate matrix $P_{\mathbf{B}}$ associated to the basis \mathbf{B} is $P_{\mathbf{B}} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$. The inverse of $P_{\mathbf{B}}$ is $P_{\mathbf{B}}^{-1} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$. It follows that

$$[\vec{x}]_{\mathbf{B}} = P_{\mathbf{B}}^{-1}\vec{x} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}.$$

(b) Find a basis for the column space $\text{Col}A$ of the matrix A where

$$A = \begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 5 & -8 & 7 & 1 \end{bmatrix}.$$

What is the dimension of $\text{Col}A$?

To find a basis for $\text{Col}A$, we only need to reduce A to a matrix in row echelon form. Note that

$$A = \begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 5 & -8 & 7 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & -1/2 & 2 & -3/2 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 5/2 \end{bmatrix}.$$

Thus the first, second and the fourth columns of A contain pivots so the vectors

$$\begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -8 \end{bmatrix}, \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$$

form a basis for $\text{Col}A$ and thus $\dim \text{Col}A = 3$.