Math 205 A & B Quiz 05 ver01 page 1 02/28/2014 **Name** Suggested Solution

1A. Let V be a vector space and let H be a subset of V. What three conditions must H satisfy in order to be a subspace of V?

(1) $\vec{0} \in H$

(2) If $\vec{u} \in H$ and $\vec{v} \in H$ then $\vec{u}+\vec{v} \in H$

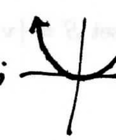
(3) If $\vec{u} \in H$ and c is a scalar, then $c\vec{u} \in H$.

1B. Recall $\mathbf{F}$ is the vector space of all continuous functions $f : \mathbb{R} \to \mathbb{R}$. Let H be the subset of $\mathbf{F}$ consisting of all functions f whose graphs have no points above the x axis. (So the graph of any member of H may have points right on the x axis and/or below it).

Which parts of the definition of subspace (see (1A)) does this H satisfy? Which parts does it fail? Explain your answers! (You may use good pictures).

(1) In $\mathbb{F}$, the zero vector $\vec{0}$ is the constant function $f(x)=0$ for all x; its graph is the x-axis and therefore has no points above the x-axis.

(2) Suppose $\vec{u}$ and $\vec{v}$ belong to H. So $\vec{u}$ is some function $g \in \mathbb{F}$ and satisfying $g(x) \leq 0$ for all x; this is another way to say the graph of g has no points above the x-axis. Also, $\vec{v}$ is some function q satisfying $q(x) \leq 0$ for all x's. Now consider $\vec{u}+\vec{v}$. It's the function $g+q$, and its value at x is $(g+q)(x) = g(x)+q(x)$. But since $g(x) \leq 0$ and $q(x) \leq 0$, the sum $g(x)+q(x) \leq 0$ also. That is, the graph of $\vec{u}+\vec{v}$ never goes above the x-axis and hence $\vec{u}+\vec{v} \in H$.

(2) H fails this part of the definition and here's a counter example:
let $\vec{u}$ be the function $-(x^2)$; its graph is  and so $\vec{u} \in H$.
But taking $c=-1$, the function $c\vec{u}$
is the function $(-1)(-(x^2)) = x^2$, and ITS graph is 
which is NOT in H since it has points above the x-axis

THIS QUIZ CONTINUES ON THE OTHER SIDE!

2. Consider these members of $\mathbb{P}_3$: $\mathbf{v}_1 = 5x^3 + 2x^2$, $\mathbf{v}_2 = 6x^3 + x + 3$, $\mathbf{v}_3 = 3x^3 + 6x^2 - 2x - 6$, $\mathbf{v}_4 = x^3 + 2$.

2A: Suppose A, B, C, and D are four constants. What augmented matrix represents finding all solutions α, β, γ and δ of the vector equation

$$\alpha\mathbf{v}_1 + \beta\mathbf{v}_2 + \gamma\mathbf{v}_3 + \delta\mathbf{v}_4 = Ax^3 + Bx^2 + Cx + D?$$

By matching the coefficients of like-powers of x on both sides of the "=" sign we see:

$$\begin{cases} \alpha 5 + \beta 6 + \gamma 3 + \delta 1 = A \\ \alpha 2 + \beta\cdot 0 + \gamma 6 + \delta\cdot 0 = B \\ \alpha\cdot 0 + \beta\cdot 1 + \gamma(-2) + \delta\cdot 0 = C \\ \alpha\cdot 0 + \beta\cdot 3 + \gamma(-6) + \delta\cdot 2 = D \end{cases}$$

the corresponding matrix is $\left[\begin{array}{cccc|c} 5 & 6 & 3 & 1 & A \\ 2 & 0 & 6 & 0 & B \\ 0 & 1 & -2 & 0 & C \\ 0 & 3 & -6 & 2 & D \end{array}\right]$

2B: If $A = B = C = D = 0$, then $Ax^3 + Bx^2 + Cx + D$ is zero-vector in $\mathbb{P}_3$. Use the answer to 2A to find a non-trivial way to write the zero-vector as a linear combination of the four vectors $\mathbf{v}_1, \ldots, \mathbf{v}_4$. (Explicitly give your values of α, β, γ and δ. BONUS: show that your answers "works".

we need the rref of $\left[\begin{array}{cccc|c} 5 & 6 & 3 & 1 & 0 \\ 2 & 0 & 6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 2 & 0 \end{array}\right]$ which is $\begin{array}{c} \begin{array}{ccccc} \alpha & \beta & \gamma & \delta & \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$

The solutions of the system thus represented are

$$\begin{cases} \alpha = -3\gamma \\ \beta = 2\gamma \\ \gamma \text{ is free} \\ \delta = 0 \end{cases} \quad \text{or} \quad \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{bmatrix} = \gamma \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} \text{ where } \gamma \text{ is free.}$$

to obtain a nontrivial solution, let's pick $\gamma = 1$ to get $\begin{cases} \alpha = -3 \\ \beta = 2 \\ \gamma = 1 \\ \delta = 0 \end{cases}$

Let's check:

does $-3\vec{v}_1 + 2\vec{v}_2 + 1\vec{v}_3 + 0\vec{v}_4 = \vec{0}$?

does

$$-3(5x^3 + 2x^2) + 2(6x^3 + x + 3) + (3x^3 + 6x^2 - 2x - 6) + 0(x^3 + 2) \overset{?}{=} 0x^3 + 0x^2 + 0x + 0$$

does

$$-15x^3 - 6x^2 + 12x^3 + 2x + 6 + 3x^3 + 6x^2 - 2x - 6 \overset{?}{=} 0x^3 + 0x^2 + 0x + 0$$

does

$$\underbrace{(-15+12+3)}_{0}x^3 + \underbrace{(-6+6)}_{0}x^2 + \underbrace{(2-2)}_{0}x + \underbrace{(6-6)}_{0} \overset{?}{=} \vec{0}$$

YES

2C. Is the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ linearly independent? Explain.