1A. Let V be a vector space and let H be a subset of V. What three conditions must H satisfy in order to be a subspace of V?

1B. Recall $\mathbf{F}$ is the vector space of all continuous functions $f : \mathbb{R} \to \mathbb{R}$. Let H be the subset of $\mathbf{F}$ consisting of all functions f whose graphs have no points above the x axis. (So the graph of any member of H may have points right on the x axis and/or below it).

Which parts of the definition of subspace (see (1A)) does this H satisfy? Which parts does it fail? Explain your answers! (You may use good pictures).

THIS QUIZ CONTINUES ON THE OTHER SIDE!

2. Consider these members of $\mathbb{P}_3$: $\mathbf{v}_1 = 5\,x^3 + 2\,x^2$, $\mathbf{v}_2 = 6\,x^3 + x + 3$, $\mathbf{v}_3 = 3\,x^3 + 6\,x^2 - 2\,x - 6$, $\mathbf{v}_4 = x^3 + 2$.

2A: Suppose A, B, C, and D are four constants. What augmented matrix represents finding all solutions α, β, γ and δ of the vector equation

$$\alpha\mathbf{v}_1 + \beta\mathbf{v}_2 + \gamma\mathbf{v}_3 + \delta\mathbf{v}_4 = Ax^3 + Bx^2 + Cx + D?$$

2B: If $A = B = C = D = 0$, then $Ax^3 + Bx^2 + Cx + D$ is zero-vector in $\mathbb{P}_3$. Use the answer to 2A to find a non-trivial way to write the zero-vector as a linear combination of the four vectors $\mathbf{v}_1, \ldots, \mathbf{v}_4$. (Explicitly give your values of α, β, γ and δ. BONUS: show that your answers "works".

2C. Is the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ linearly independent? Explain.