1A. What is the general formula for the inverse of the 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$; under what condition(s) does it exist?

The inverse is $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ provided that $ad-bc \neq 0$; otherwise there is no inverse.

1B. Let $M = \begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix}$. Find the inverse M^{-1} of M. Write the answer in the form $\begin{bmatrix} q & r \\ s & t \end{bmatrix}$ (some of the entries might be fractions).

$$M^{-1} = \frac{1}{5\cdot4-3\cdot6}\begin{bmatrix} 4 & -6 \\ -3 & 5 \end{bmatrix} = \frac{1}{20-18}\begin{bmatrix} 4 & -6 \\ -3 & 5 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 4 & -6 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -3/2 & 5/2 \end{bmatrix}$$

2. Suppose $A = \begin{bmatrix} p & 5 & 1 \\ 3 & w & 0 \\ 7 & 6 & 3 \\ 1 & 4 & 10 \end{bmatrix}$, $B = \begin{bmatrix} 10 & 8 & 5 \\ k & 4 & 9 \\ 3 & 7 & 2 \end{bmatrix}$, and their product AB is $C = \begin{bmatrix} 88 & a & 77 \\ c & v & 33 \\ 109 & g & 95 \\ f & 94 & u \end{bmatrix}$.

Find g, w, p, k. (For each of those unknowns g, w, p, k you should be able to find an equation involving just that one unknown; no systems required!)

g is in row 3, col 2 of the product, so it's "row 3 of A $\times$ col 2 of B"

$= \text{"}\begin{bmatrix} 7 & 6 & 3 \end{bmatrix}\begin{bmatrix} 8 \\ 4 \\ 7 \end{bmatrix}\text{"} = 7\cdot8 + 6\cdot4 + 3\cdot7 = 56+24+21 = \boxed{101 \leftarrow g}$

W is in row 2 of A, and multiplying this row by column 3 of B gives the 2,3 entry of C which is 33 AND this involves only the unknown w.

So we find "$\begin{bmatrix} 3 & w & 0 \end{bmatrix}\begin{bmatrix} 5 \\ 9 \\ 2 \end{bmatrix}$" $= 15 + 9w + 0 = 33 \Rightarrow 9w = 18 \Rightarrow \boxed{w=2}$

using "$\begin{bmatrix} p & 5 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 9 \\ 2 \end{bmatrix} = 77$" we get $5p + 45 + 2 = 77$, so $5p = 30$, so $\boxed{p=6}$

finally, "$\begin{bmatrix} 7 & 6 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ K \\ 3 \end{bmatrix} = 109$" tells us $70 + 6K + 9 = 109$, so $6K = 30$, so $\boxed{K=5}$

(you can check that using other rows & columns in which these unknowns appear give the same results)

3. What is the transpose of the matrix B from problem 2?

$$\begin{bmatrix} 10 & K & 3 \\ 8 & 4 & 7 \\ 5 & 9 & 2 \end{bmatrix}$$

4. Let A be as in problem 2; you do not need to know the actual entries to answer this question.

4a) You can compute the product $A\,A^T$ (the dimensions are right). What are the dimensions of the resulting product, and how many individual pairs of numbers will need to be multiplied in order to compute $A\,A^T$?

A is 4×3 and A^T is 3×4 $\therefore$ their product $A\,A^T$ has 4 rows & 4 columns.

Each of these 16 elements requires multiplying 3 elements from a row of A by 3 corresponding elements from a column of A^T; that is, each entry of AA^T requires 3 multiplications, so the total is $16\times3 = \underline{\underline{48 \text{ pairs}}}$

4b) Similarly, the numbers of rows and columns are right to make it possible to find $A^T\,A$. What is the size of this matrix, and again, how many individual pairs of numbers will need to be multiplied in order to compute $A^T\,A$?

The product A^TA is the result of multiplying a 3×4 by a 4×3, giving a 3×3 product matrix.

Each row of A^T and each col. of A has 4 entries, so multiplying one row of A^T by a col. of A multiplies 4 pairs of #s. Since A^TA has 9 entries, we'll need to multiply a total of

$9\times4 = \underline{\underline{36 \text{ pairs}}}$.