MATH 205A,B - LINEAR ALGEBRA
WINTER 2013

QUIZ 6

NAME: **Section:**(Circle one) A(1 : 10) B(2 : 40)

Show ALL your work CAREFULLY.

(a) Find an explicit description of the null space Nul$A$, by listing vectors that span the null space.

$$A = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 1 & -1 & 3 & -1 \\ 0 & 1 & 2 & 0 \end{bmatrix}.$$

The nullspace Nul$A$ **is the solutions to the equation** $A\vec{x} = \vec{0}$**. A straightforward calculation shows that the row reduced echelon form of** A **is**

$$\begin{bmatrix} 1 & 0 & 0 & 1/4 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & -1/4 \end{bmatrix}.$$

It follows that the solutions are of the form

$$\vec{x} = x_4 \begin{bmatrix} -1/4 \\ -1/2 \\ 1/4 \\ 1 \end{bmatrix} \textbf{ or equivalently } \text{Nul}A = \left\{ x_4 \begin{bmatrix} -1/4 \\ -1/2 \\ 1/4 \\ 1 \end{bmatrix} \;:\; x_4 \textbf{ in } \mathbb{R} \right\}.$$

(b) Let $M_{2\times 2}$ be the vector space of all 2×2 matrices with real entries. Define $T : M_{2\times 2} \to M_{2\times 2}$ by $T(A) = A + A^T$ where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Show that T is a linear transformation. [A vector here is a 2×2 matrix.]

By definition,

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} 2a & b+c \\ b+c & 2d \end{bmatrix}.$$

For any real number k**,**

$$T\left(k\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = T\left(\begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}\right) = \begin{bmatrix} 2ka & kb+kc \\ kb+kc & 2kd \end{bmatrix} = kT\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right).$$

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Similarly,

$$\begin{aligned}
T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix}\right) &= T\left(\begin{bmatrix} a+a' & b+b' \\ c+c' & d+d' \end{bmatrix}\right) \\
&= \begin{bmatrix} 2(a+a') & (b+b')+(c+c') \\ (b+b')+(c+c') & 2(d+d') \end{bmatrix} \\
&= \begin{bmatrix} 2a & b+c \\ b+c & 2d \end{bmatrix} + \begin{bmatrix} 2a' & b'+c' \\ b'+c' & 2d' \end{bmatrix} \\
&= T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) + T\left(\begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix}\right).
\end{aligned}$$

Hence, T is a linear transformation.