1a. Let $P = \begin{bmatrix} s & t \\ g & h \end{bmatrix}$. In terms of s, t, g and h, what is the inverse of P? And what condition needs to be satisfied for this matrix P^{-1} to exist?

$$P^{-1} = \frac{1}{sh-gt}\begin{bmatrix} h & -t \\ -g & s \end{bmatrix}, \text{ provided that } sh-gt \neq 0.$$

1b. Let $Q = \begin{bmatrix} 8 & 6 \\ 7 & 5 \end{bmatrix}$. Use the answer to (1a) to find Q^{-1}; simplify all entries as much as possible.

$$Q^{-1} = \frac{1}{8\cdot 5 - 7\cdot 6}\begin{bmatrix} 5 & -6 \\ -7 & 8 \end{bmatrix} = \frac{-1}{2}\begin{bmatrix} 5 & -6 \\ -7 & 8 \end{bmatrix} = \begin{bmatrix} -5/2 & 3 \\ 7/2 & -4 \end{bmatrix}$$

2. Let $D = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and let $E = \begin{bmatrix} q & r & s \\ t & u & v \\ m & n & z \end{bmatrix}$.

2A) What is the entry in the third row, second column of the product DE?

$=$ "third row of D × second col. of E" $= gr + hu + in$

2B) What is the entry in the third row, second column of the product ED?

$=$ "third row of E × second col of D" $= mb + ne + zh$

3. Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation and the associated matrix is M.

Use the following statements in the multiple choice question below.

A) $T(\mathbf{x}) = \mathbf{w}$ has at least one solution $\mathbf{x}$ in $\mathbb{R}^n$ for every $\mathbf{w}$ in $\mathbb{R}^m$.

B) The matrix M has more rows than columns.

C) $T(\mathbf{z}) = T(\mathbf{w})$ implies $\mathbf{z} = \mathbf{w}$ for all $\mathbf{z}$ and $\mathbf{w}$ in $\mathbb{R}^n$.

D) The matrix M has at least one non-pivot column.

E) The matrix M has a row of 0's in its RREF.

F) For every $\mathbf{b} \in \mathbb{R}^m$ there is some $\mathbf{a} \in \mathbb{R}^n$ for which $T(\mathbf{a}) = \mathbf{b}$.

G) There are elements of the codomain of T which are not in the range of T.

H) $T(\mathbf{p}) = \mathbf{q}$ has at most one solution $\mathbf{p}$ in $\mathbb{R}^n$ for every $\mathbf{q}$ in $\mathbb{R}^m$.

P) If $\mathbf{a} = \mathbf{z}$, then $T(\mathbf{a}) = T(\mathbf{z})$, for all $\mathbf{a}$ and $\mathbf{z}$ in $\mathbb{R}^n$.

Q) $M\mathbf{x} = \mathbf{c}$ is consistent for every $\mathbf{c}$ in $\mathbb{R}^m$.

R) There are no free variables in the equation $M\mathbf{x} = \mathbf{c}$ for any $\mathbf{c}$ in $\mathbb{R}^m$.

S) The columns of M form a linearly independent set of vectors.

Answer these four **multiple-choice** questions:

3.1: Which of the above statements either is a definition of "T is one-to-one" or can be used to prove T is 1-1? List all qualifying statements by letter: C H R S

3.2: Which of the above statements either is a definition of "T is onto $\mathbb{R}^m$" or can be used to prove T is onto $\mathbb{R}^m$? List all qualifying statements by letter: A F Q

3.3: Which statements, if any, can be used to show T is *not* 1-1? List by letter:

D

3.4: Which, if any, can be used to show T is not onto $\mathbb{R}^m$?

B E G