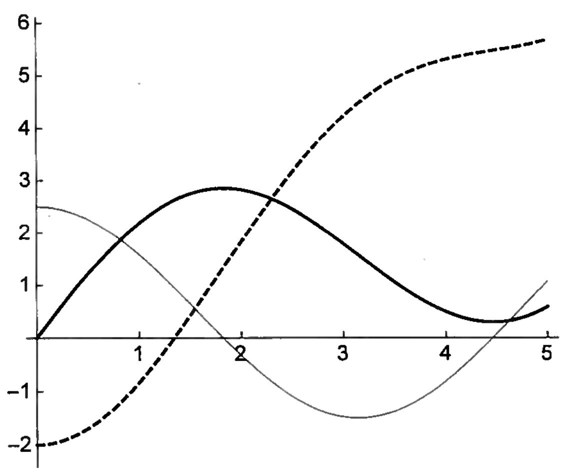
Calculus I
MATH 105

EXAM 1
February 13, 2015

Name: KEY

SHOW ALL WORK, CLEARLY AND LEGIBLY, TO RECEIVE FULL CREDIT. CORRECT SPELLING, ORGANIZATION OF YOUR SOLUTION, AND PROPER USE OF MATHEMATICAL NOTATION ALL COUNT. YOU MAY USE A STAND-ALONE GRAPHING CALCULATOR, BUT NOT ANY INTERNET-BASED CALCULATORS. NO NOTES, BOOKS, OR OTHER ADDITIONAL RESOURCES ARE PERMITTED. GOOD LUCK!

1.) (10 pts.) The graphs below are f, f', and f''. State which is which, and explain how you know this.



Dashed line: f

Bold line: f'

Thinner line: f''

f increases throughout, so its slopes are always positive

f' is always positive.

f' has slope 0 at about $\underbrace{x = 18 \text{ and } x = 4.3}$

$f'' = 0$ at those x-values

No curve has slope 0 at about $x = 1.3$, where the dashed line crosses the x-axis, so the dashed line cannot be the derivative of either other graph.

2.) (15 pts.)

a.) (5 pts.) Suppose $\lim_{x\to 5^-} f(x) = 2$ and $\lim_{x\to 5^+} f(x) = 4$. Is it possible that $\lim_{x\to 5} f(x) = 3$? Justify your answer.

No. For $\lim_{x\to 5} f(x)$ to exist at all, we would need $\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x)$, and this does not happen, so $\lim_{x\to 5} f(x)$ D.N.E.

b.) (5 pts.) Suppose $g(x) = \dfrac{x^2+3x-10}{x-2}$. What is $g(2)$? [Note: $g(x)$ is not related to $f(x)$ in part (a).]

$$g(2) = \frac{2^2+3\cdot 2-10}{2-2} = \frac{0}{0} \longrightarrow \boxed{\text{D.N.E.}}$$

c.) (5 pts.) What is $\lim_{x\to 2} \dfrac{x^2+3x-10}{x-2}$?

$$= \lim_{x\to 2} \frac{(x-2)(x+5)}{x-2}$$

$$= \lim_{x\to 2} (x+5)$$

$$= \boxed{7}$$

3.) (15 pts.)

a.) (5 pts.) Give an example of a polynomial, and describe in words what it means for a function to be a polynomial.

$$f(x) = 7x^6 - 3x^4 - 2$$

Real-valued coefficients, x raised to whole number exponents, terms added and/or subtracted together.

b.) (5 pts.) Give an example of a rational function, and describe in words what it means for a function to be a rational function.

$$f(x) = \frac{2x^2 - 5}{9x^{100} + 3x}$$

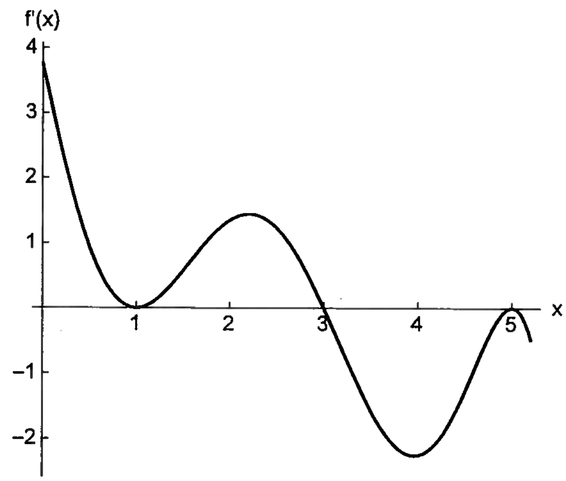
polynomial divided by a polynomial

c.) (5 pts.) Give an example of an exponential function, and describe in words what it means for a function to be an exponential function.

$$y = 5^x$$

A constant base and a variable in the exponent

4.) (15 pts.) Shown below is a graph of f' on its entire domain. The graph is NOT f.



a.) (3 pts.) At which x-value(s) does f have a stationary point?

Where $f' = 0$:

$x = 1, 3, 5$

b.) (3 pts.) At which x-value(s) does f' have a stationary point?

At $x \approx 1, 2, 4, 5$

c.) (3 pts.) At which x-value(s) is f greatest?

When f' switches from positive to negative, so f switches from increasing to decreasing at its local max:

$x = 3$

d.) (3 pts.) At which x-value(s) is f increasing?

Where $f' > 0$: $x \in [0, 3]$

e.) (3 pts.) At which x-value(s) is f concave up?

Where f' is increasing:

$x \in [1, 2] \cup x \in [4, 5]$

5.) (15 pts.) For each of the following questions, let $f(x) = \sqrt{x} + \frac{1}{x^3}$. On this page, you may complete the exercises using the Power Rule we learned for computing derivatives and antiderivatives.

$$f(x) = x^{\frac{1}{2}} + x^{-3}$$

a.) (5 pts.) Compute the general antiderivative $F(x)$.

$$F(x) = \frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1} + \frac{1}{-3+1} x^{-3+1} + C$$

$$F(x) = \frac{2}{3} x^{\frac{3}{2}} - \frac{1}{2} x^{-2} + C$$

b.) (5 pts.) Solve the initial value problem in which the differential equation is $f(x)$ and the initial condition is $F(1) = 3$.

$$F(1) = \frac{2}{3}(1)^{3/2} - \frac{1}{2}(1)^{-2} + C = 3$$

$$\frac{2}{3} - \frac{1}{2} + C = 3$$

$$\frac{1}{6} + C = 3$$

$$C = 2\frac{5}{6}$$

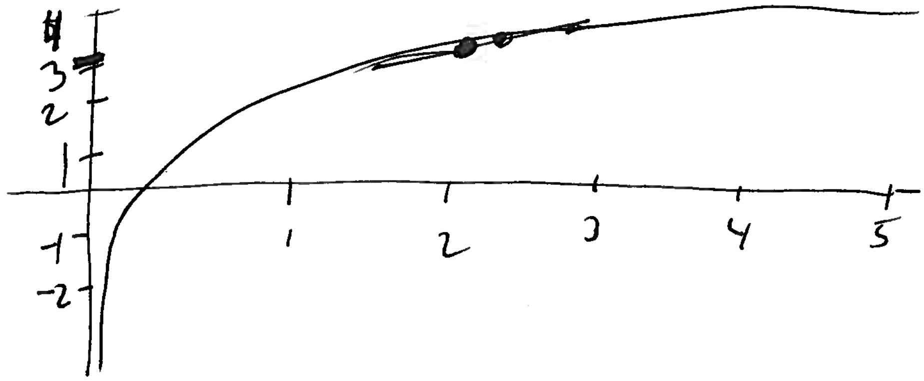
$$F(x) = \frac{2}{3}x^{3/2} - \frac{1}{2x^2} + \frac{17}{6}$$

c.) (5 pts.) Compute $f'(x)$.

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 3x^{-4}$$

6.) (15 pts.) Consider the function $f(x) = \ln(8x)$.

a.) (5 pts.) Draw $f(x)$, showing the graph for x-values ranging from 0 to 5.



b.) (5 pts.) Numerically zoom to estimate $f'(2)$.

$f(2) = 2.77$

$f(2.1) = 2.82$

$$f'(2) \approx \frac{f(2.1) - f(2.0)}{2.1 - 2.0} = \frac{2.82 - 2.77}{.1} = \boxed{.5}$$

c.) (5 pts.) Explain, referring to your graph, how the idea of numerical zooming leads us to the exact definition of the derivative at a point (such as at the point $x = 2$).

$f(x) = f(2)$

$f(x+h)$ is our $f(2.1)$ with $h = .1$

The derivative at a point moves h infinitely close to 0 to compute the exact tangent line slope. Otherwise we are approximating, using a secant line slope.

7.) (15 pts.) Use **the limit definition of the derivative** to compute $f'(x)$ for $f(x) = 3x^2 + 5x$. *[NOTE: you may use the Power Rule to check your result, but that alone will earn you no credit.]*

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[3(x+h)^2 + 5(x+h)] - [3x^2 + 5x]}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + 5x + 5h - 3x^2 - 5x}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2 + 5h}{h}$$

$$= \lim_{h \to 0} \frac{h(6x + 3h + 5)}{h}$$

$$= \lim_{h \to 0} (6x + 3h + 5)$$

$$= 6x + 5$$