MATH 205A,B - LINEAR ALGEBRA
WINTER 2013

QUIZ 5

NAME: **Section:**(Circle one) A(1 : 10) B(2 : 40)

Show ALL your work CAREFULLY.

(a) Suppose $\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 3$. Find $\det\begin{bmatrix} a & d & g \\ 2b & 2e & 2h \\ c & f & i \end{bmatrix}$.

$$\det\begin{bmatrix} a & d & g \\ 2b & 2e & 2h \\ c & f & i \end{bmatrix} = 2\cdot\det\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} = 2\cdot\det\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}^T = 2\cdot\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 6.$$

(b) Use row operations to find $\det A$ where

$$A = \begin{bmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{bmatrix}.$$

Is A invertible?

First note that if $a = b$ or $a = c$ then A will have two identical rows and thus is row equivalent to a matrix that has a row of zeros. In such cases, $\det A = 0$ and A is not invertible. Now, assume that $a \neq b$ and $a \neq c$. Then we have

$$A \sim \begin{bmatrix} 1 & a & b+c \\ 0 & b-a & a-b \\ 1 & c & a+b \end{bmatrix} \sim \begin{bmatrix} 1 & a & b+c \\ 0 & b-a & a-b \\ 0 & c-a & a-c \end{bmatrix} \sim \begin{bmatrix} 1 & a & b+c \\ 0 & 1 & -1 \\ 0 & c-a & a-c \end{bmatrix} \sim \begin{bmatrix} 1 & a & b+c \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & a & b+c \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}.$$

It follows that $\det A = 0$ and hence A is not invertible.

Date: February 13, 2013.