Name:_____

Exam 1 - Math 105

Show all your work to receive full credit for a problem. There are a total of 72 points on this test. Good luck!

1. (4 points) The following data describe the growth of the population P (in thousands) of Gotham City during a 10-year period. Estimate the rate of growth in 1989.

Year	1984	1986	1988	1990	1992	1994
P	265	293	324	358	395	427

2. (6 points) Find the equation of the line tangent to the graph of $g(x) = \frac{3}{x^2} - \frac{4}{x^3}$ at $x = -1$.

3. (8 points) A particle moves along a straight path with acceleration function $a(t) = 15\sqrt{t}$, initial position $x(0) = 5$, and initial velocity $v(0) = 7$. Find the particle's position function $x(t)$.

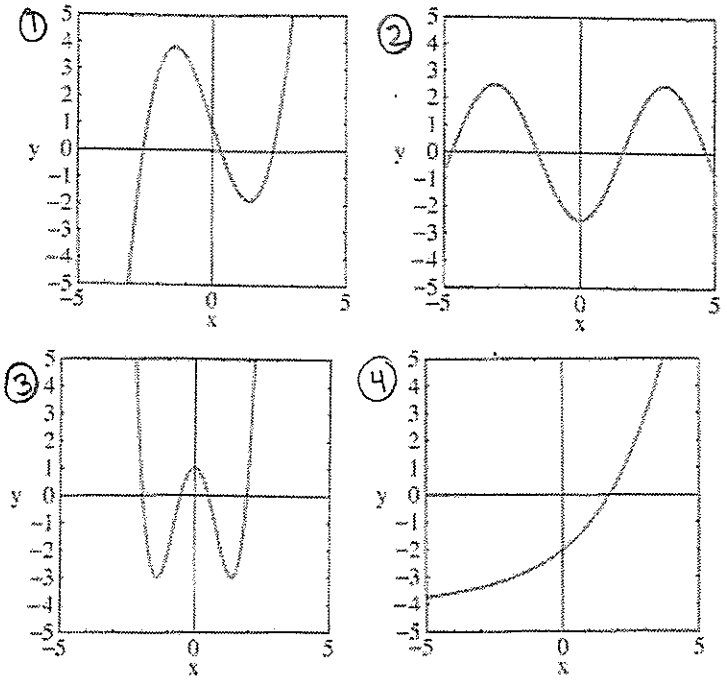
4. (8 points) Use the formal definition of the derivative to find $f'(x)$ when $f(x) = x^2 + 5$.

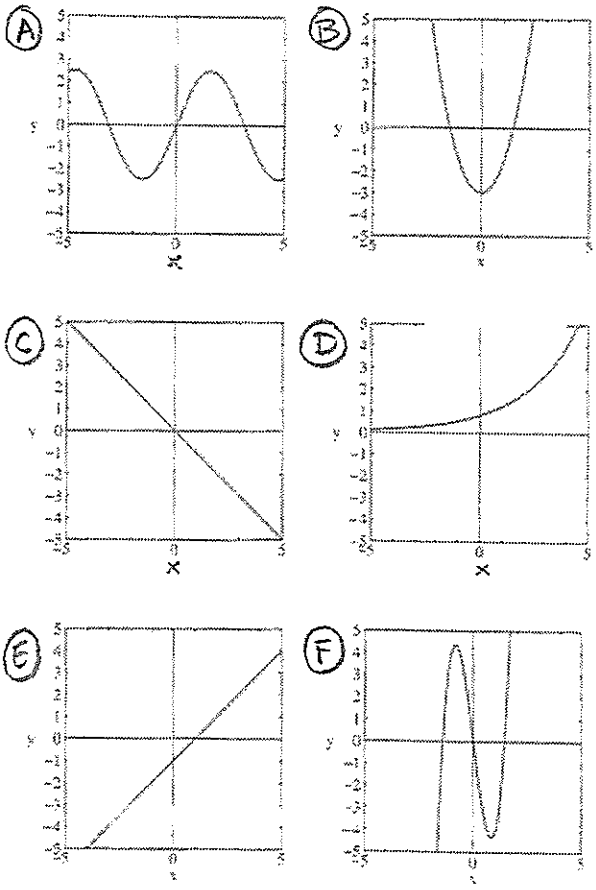
5. (8 points) Show that $y = x^2 - x^{-1}$ is a solution of the differential equation $y'' - \dfrac{2y}{x^2} = 0$.

6. (8 points) Match each of the the following graphs (on the left) to its derivative (pictured on the right). (Note that there are more graphs on the right than on the left, so not all of the graphs will be matched.)

f

f'





7. (3 points each, 15 points total.) The following statements are either true or false. If True, write down a short explanation for your answer, and if False, give an example or explanation of why it doesn't always hold true.

   (a) If $\lim_{x \to c} f(x) = 10$ then $f(c) = 10$.

   (b) Suppose an object is moving in a straight path with position function $s(t)$. If $s(t)$ is positive and decreasing, then the velocity $v(t)$ is negative.

   (c) The derivative of $f(x) = |x|$ at $(0, 0)$ does not exist.

   (d) If $h(x) = 2x^3 - 7x^2 + 3x - 4$ then $h'(x) = 6x^2 - 14x + 3 - 4$.

   (e) Suppose that $f(1) = -2$ and $f'(x) \leq 3$ for all x in the interval $[-10, 10]$. Then it is possible that $f(4) = 8$.

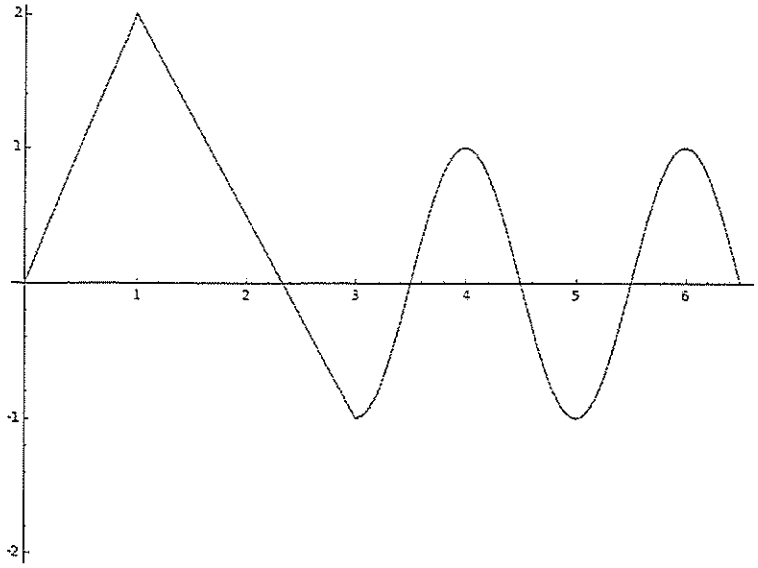


Figure 1: Graph of $g(x)$

8. (3 pts each, 15 pts total.) Let $G(x)$ be an antiderivative of $g(x)$, the function shown in Figure 1. Answer the following questions about G. (You may use estimates if you're not sure of the exact values.)

   (a) What is $G'(3)$? What is $G''(3)$?

   (b) Where are the maximum and minimum points of G?

   (c) On what intervals is G increasing? On what intervals is G decreasing?

   (d) What are the inflection points of G?

   (e) On what intervals is G concave up? On what intervals is G concave down?

9. (2 points extra credit) Tell me about something in this class that you understand really well, and what has helped you understand it. Be specific, and convince me that you understand it. (An example of something that is not a great answer: I understand how to add numbers because it's easy.)