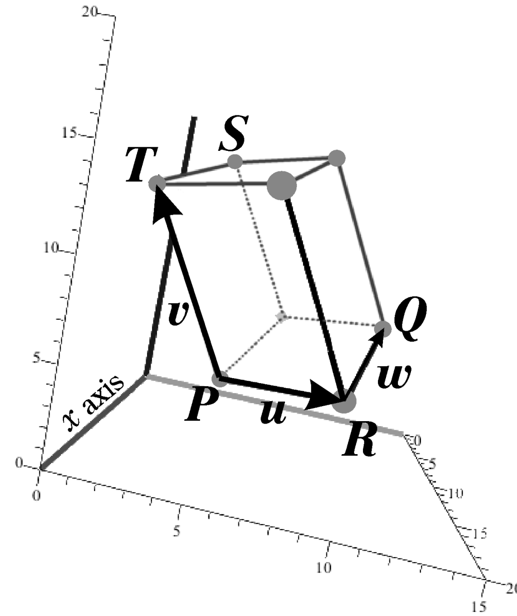
 Name____________________

1. Consider the two lines ℓ_1 and ℓ_2 in $\mathbf{R}^3$ parameterized by $\overrightarrow{(x, y, z)} = \overrightarrow{(s+3, s+4, 2s+8)}$ and $\overrightarrow{(x, y, z)} = \overrightarrow{(5, 1, z_0)} + t\overrightarrow{(1, 2, 4)}$, respectively.

1A: Find z_0 so that these two lines intersect at some point $\mathbf{p}$. Show all your work.

1B: What is $\mathbf{p}$, explicitly?

2. In the parallelepiped drawn here, the point P is $(12, 5, 4)$, the point T is $(13, 1, 14)$, the point Q is $(5, 12, 8)$, and the vector $\mathbf{u}$ is $\overrightarrow{(3, 5, 1)}$.



2A. What are the vectors $\mathbf{v}$ and $\mathbf{w}$, and the points R and S? (Label your four answers).

2B. To the nearest 1/100 of a degree, what is the angle between $\mathbf{v}$ and $\mathbf{u}$?

2C. Find the projection of $\mathbf{v}$ on $\mathbf{u}$.

2D. Find the equation of the plane containing the points P, R and T.

2E. Find the volume of this parallelepiped.

3a. Find the center of the ellipsoid E whose equation is $x^2 + 36\,y^2 + 1008 + 4\,z^2 + 48\,z = 360\,y$

3b. Find the three points A, B, C on E with the maximum x, y, and z coordinates, respectively. Label your answers.

3c. Find the equation of the plane tangent to E at the point C.

3d. Find the equation of the plane tangent to E at the point B.

4. Three level sets are plotted below for the function $z = f(x, y) = -(x+1)^2 + 4(y-3)^2 + 5$

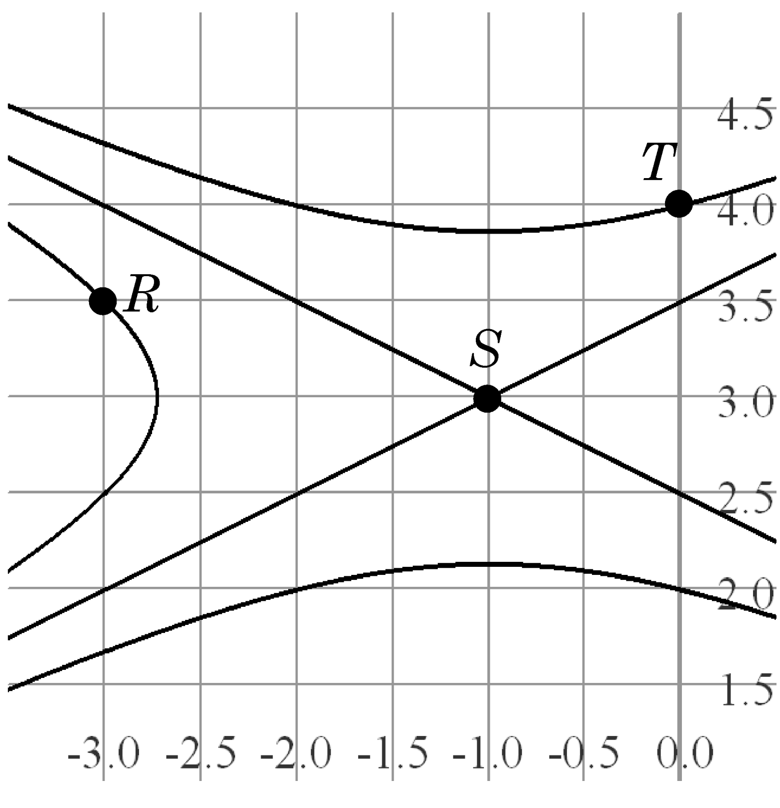
Find the equations of the three level curves through the points R, S, and T; show your work and answers in the spaces below:

(4A) Your work and answer for R: (express x as a function of y in your answer)

(4B) Your work and answer for S:

(4C) Your work and answer for T: (express y as a function of x in your answer)

(4D) What is the name given to the surface given by the graph of f?



5. Suppose f is a function from some subset of $\mathbf{R}^n$ to $\mathbf{R}$. In class we defined the partial derivative of f with respect to the i^{th} variable x_i.

5A. Explicitly write that definition out for $\partial f/\partial y$ in the case $f(x, y, z) : \mathbf{R}^3 \to \mathbf{R}$.

5B. Show how to find $\partial f/\partial y$ by the definition for $f(x, y, z) = y\sin(x^2) + x^2y^3z$. Show all your steps.

5C. Use basic rules to find each of the following partial derivatives:

5Ca. $\dfrac{\partial}{\partial x}(\dfrac{\partial f}{\partial y})$

5Cb. $f_{z\,z}$

6A. Suppose $f : \mathbf{R}^2 \to \mathbf{R}$ is a function of two variables. Give the $\epsilon - \delta$ definition of what it means to say that

$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$

.

6B. Handed out with this exam is a plot of level curves for $f(x,y) = x^3 + y^2x - 2xy$. The thin level curves are plotted at c values which are multiples of 0.1 and the thick lines, for c at multiples of 0.5. Let $(a,b) = (1,1)$; it's marked on the graph with a gray dot. (so is the origin, so be careful). Let $L = \lim_{(x,y)\to(1,1)} f(x,y)$. What is L? To within 0.025, find the biggest possible δ corresponding to $\epsilon = 1/2$, where ϵ and δ are as in the correct limit definition.

6C. Repeat part 6B if (a,b) is the origin.

7a. An object moves along the path given by $\mathbf{f}(t) = (t^2/12)\mathbf{i} + 3t\mathbf{j} + (t^4/360)\mathbf{k}$. At $t = 3$ the object leaves the path and moves along the line tangent to its velocity vector with a constant speed equal to its speed at the instant it leaves the path. Where is the object at $t = 7$? SHOW ALL YOUR WORK.

7b. If it continues along this tangent line in this way, when will it "break through" the plane $z = 100$?