

**Math 105: Review for Exam I**

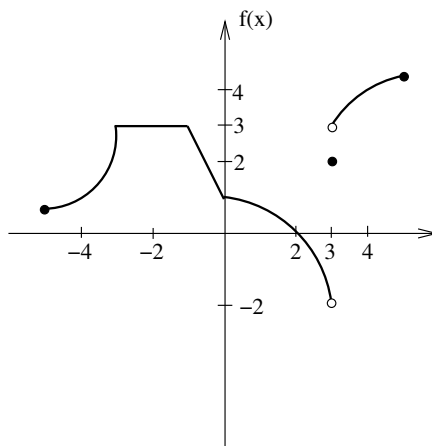
1. Let  $f(x) = 3 + \sqrt{x+5}$ .

- (a) What is the natural domain of  $f$ ?
- (b) What is the range of  $f$ ?

2. For the graph of  $f$  shown, answer the following.

(a) Evaluate the following.

- i.  $f'(-2)$
- ii.  $f(3)$
- iii.  $\lim_{x \rightarrow 3^-} f(x)$
- iv.  $\lim_{x \rightarrow 3^+} f(x)$
- v.  $\lim_{x \rightarrow 3} f(x)$
- vi.  $\lim_{x \rightarrow 2} f(x)$



- (b) Where is  $f$  discontinuous?
- (c) Where does  $f'$  fail to exist?

3. Let  $f(x) = 3x^2 - 2x$ .

(a) Compute the average rate of change of  $f$  on the interval  $[2, 2.1]$ .

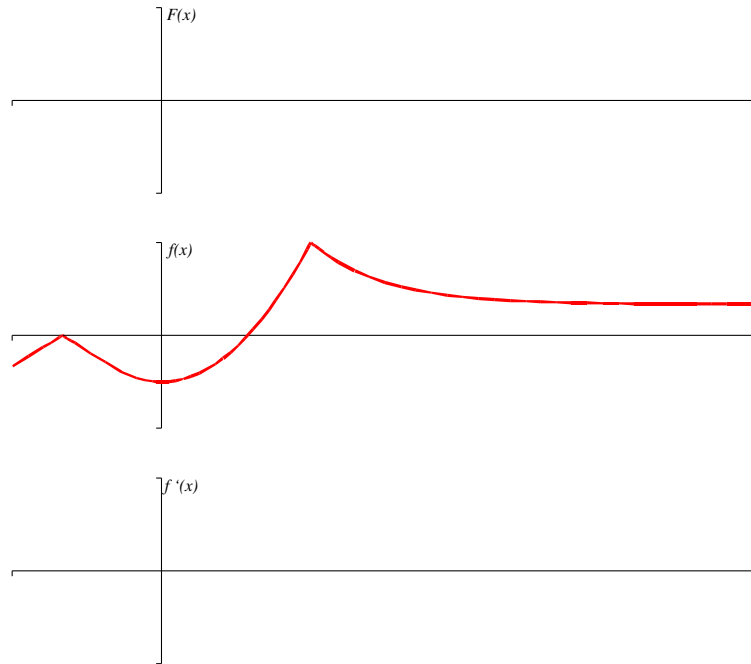
(b) Using the limit definition of the derivative, find  $f'(x)$ .

(c) Find the equation of the tangent line to  $f$  at  $x = 2$ .

(d) How would the derivative of  $g(x) = f(x) + 5$  compare to  $f'(x)$ ?

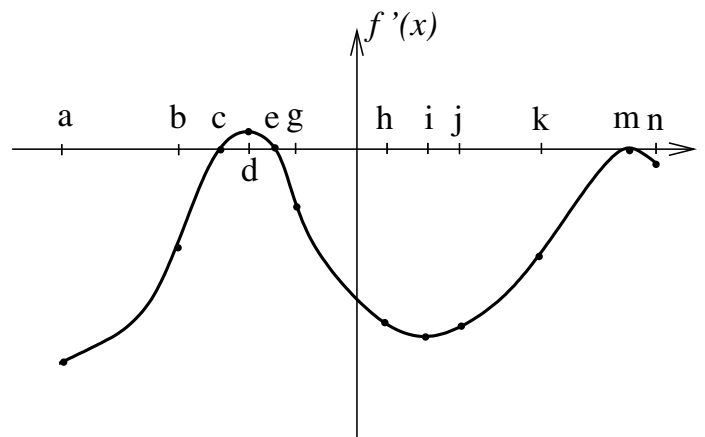
(e) How would the derivative of  $h(x) = 5f(x)$  compare to  $f'(x)$ ?

4. Given the graph of  $f$ , sketch a graph of  $f'$  and a graph of  $F$ , an antiderivative of  $f$  such that  $F(0) = -2$ .



5. Shown below is a graph of  $f'$  on its entire domain. The graph is NOT  $f$ .

- At which  $x$ -value(s)
- (a) does  $f$  have a stationary point?
  - (b) does  $f$  have a local max?
  - (c) does  $f$  have a local min?
  - (d) does  $f'$  have a stationary point?
  - (e) does  $f'$  have a local max?
  - (f) does  $f'$  have a local min?
  - (g) is  $f$  greatest?
  - (h) is  $f$  least?
  - (i) is  $f'$  greatest?
  - (j) is  $f'$  least?
  - (k) is  $f''$  greatest?
  - (l) is  $f''$  least?
- On what interval(s) is
- (a)  $f$  increasing?
  - (b)  $f$  decreasing?
  - (c)  $f'$  increasing?
  - (d)  $f'$  decreasing?
  - (e)  $f$  concave up?
  - (f)  $f$  concave down?



6. Suppose that  $T(t)$  gives the temperature in Lewiston as a function of time. In each of the following situations, determine if the signs of  $T$ ,  $T'$ , and  $T''$  are positive, negative, zero, or unknown.

(a) The temperature is 60 degrees and falling steadily.

(b) The temperature is rising more and more slowly.

(c) The temperature is  $-5$  degrees and rising.

7. The table below gives some values for a function  $f(x)$  whose derivative exists at all  $x$ .

|        |     |     |     |     |     |
|--------|-----|-----|-----|-----|-----|
| $x$    | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 |
| $f(x)$ | 5.0 | 6.2 | 7.3 | 8.2 | 9.0 |

(a) Estimate  $f'(1.05)$ .

(b) Based on the data, is  $f''(1.0)$  positive or negative?

8. Find the derivatives of the following.

(a)  $y = 2 + 3x + x^4 + 5x^6$

(b)  $y = \sqrt[6]{x} + \frac{1}{x^6} + \frac{x}{6} + \frac{6}{x} + \frac{\pi}{6} + 6^{1/2}$

9. Find antiderivatives of the following.

(a)  $y = \pi + 3x^2$

(b)  $y = 4x^5 - \frac{1}{x^6}$

10. Solve the IVP (initial value problem)  $1 = x^3 - y'(x)$  if  $y(2) = 13$ .