Calculus I **EXAM 1**
MATH 105 **February 10, 2011**

Name: KEY

YOUR GRADE IS BASED ON CORRECTNESS, COMPLETENESS, AND CLARITY ON EACH EXERCISE. EXPLAIN ALL ANSWERS COMPLETELY. YOU MAY USE A CALCULATOR, BUT NO NOTES, BOOKS, OR OTHER STUDENTS. GOOD LUCK!

1.) (10 pts.) Solve the initial-value problem $y' = 10x - 4$, $y(2) = 3$.

$y(x) = 5x^2 - 4x + C$

$3 = 5(2)^2 - 4(2) + C$

$3 = 20 - 8 + C$

$3 = 12 + C$

$C = -9$

$y = 5x^2 - 4x - 9$

2.) (15 pts.) Select ONE of the following two functions: $g(x) = \ln x$ OR $h(x) = \sin x$. For your selected function, complete the following.

a.) (3 pts.) What is the *natural domain* of your function?

$g(x)$: $(0, \infty)$

$h(x)$: all real numbers

b.) (3 pts.) What is the *range* of your function?

$g(x)$: all real numbers

$h(x)$: $[-1, 1]$

c.) (3 pts.) What (if any) are the *roots* of your function?

$g(x)$: $x = 1$

$h(x)$: $x = 0, \pm\pi, \pm 2\pi, \ldots$

d.) (3 pts.) Discuss when your function is *increasing* and when it is *decreasing.*

$g(x)$: increasing for all x in domain

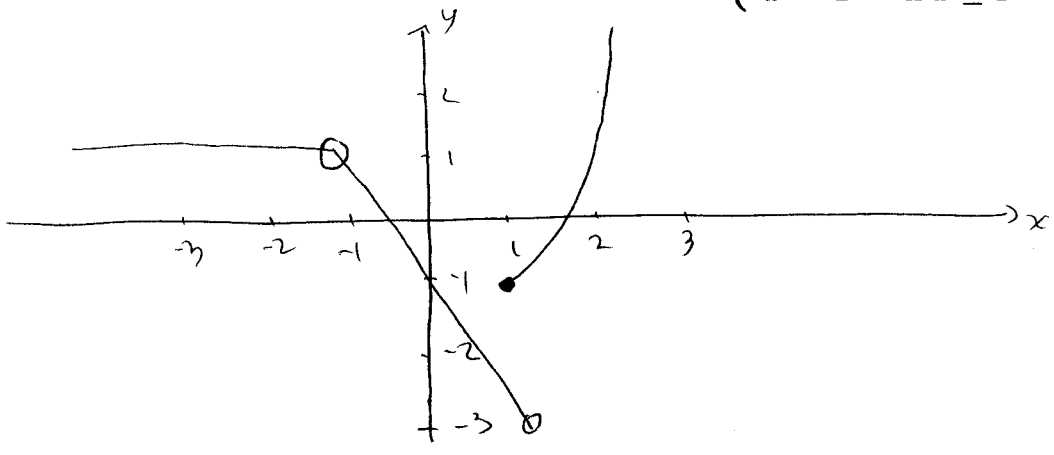
$h(x)$: increasing for $\ldots, \left(-\frac{5\pi}{2}, -\frac{3\pi}{2}\right), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right), \ldots$
and decreasing between these.

e.) (3 pts.) If your function is $g(x)$, how does the graph of $g(x+3)$ relate to your function? If your function is $h(x)$, how does the graph of $h(x+3)$ relate to your function?

In both cases: the new graph is a shift of the original function, by 3 units to the left

3.) (15 pts.)

a.) (5 pts.) Sketch the piecewise-defined function $f(x) = \begin{cases} 1 & \text{if } x < -1 \\ -2x - 1 & \text{if } -1 < x < 1 \\ x^2 - 2 & \text{if } x \geq 1 \end{cases}$.

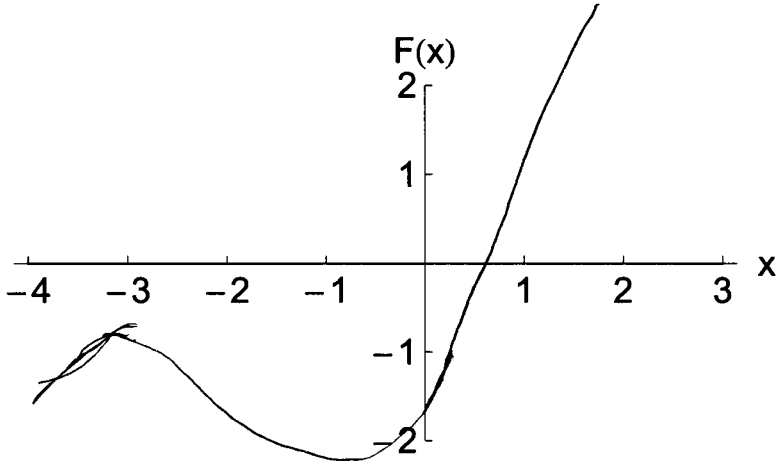


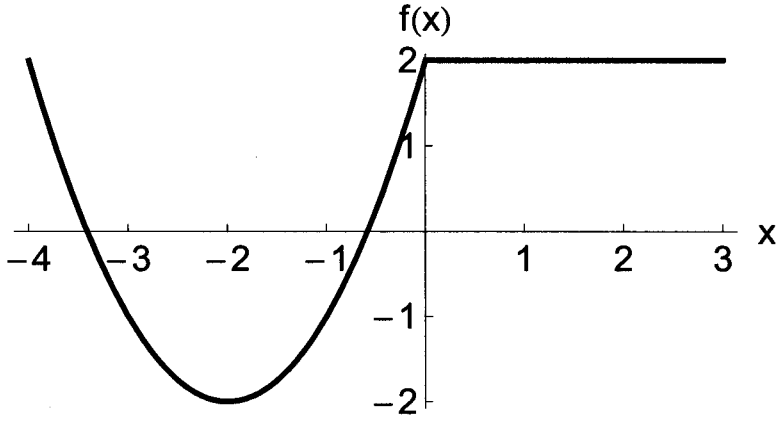
b.) (5 pts.) What is $\lim_{x \to -1} f(x)$?

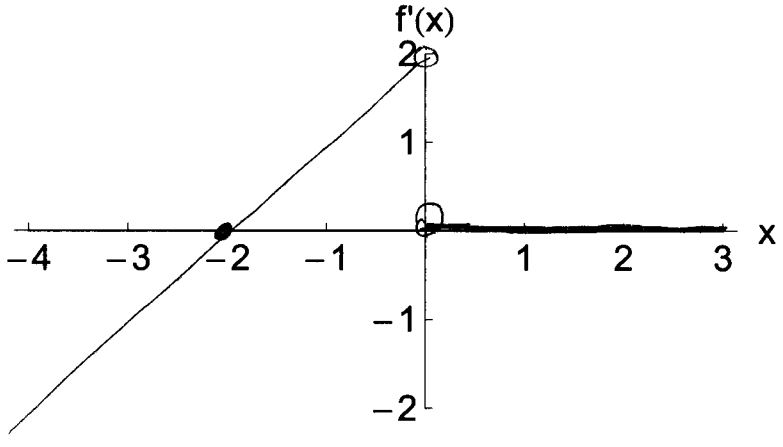
1

c.) (5 pts.) What is $\lim_{x \to 1} f(x)$? DNE

4.) (15 pts.) Given the graph of f on the middle set of axes, sketch its derivative f' on the axes below and its antiderivative F on the axes above.







5.) (15 pts.) Use **the limit definition of the derivative** to compute $f'(x)$ for $f(x) = 3 + \sqrt{x}$. *[NOTE: you may use the Power Rule to check your result, but that alone will earn you no credit.]*

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\left(3 + \sqrt{x+h}\right) - \left(3 + \sqrt{x}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right)$$

$$= \lim_{h \to 0} \frac{(x+h) - (x)}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{h}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

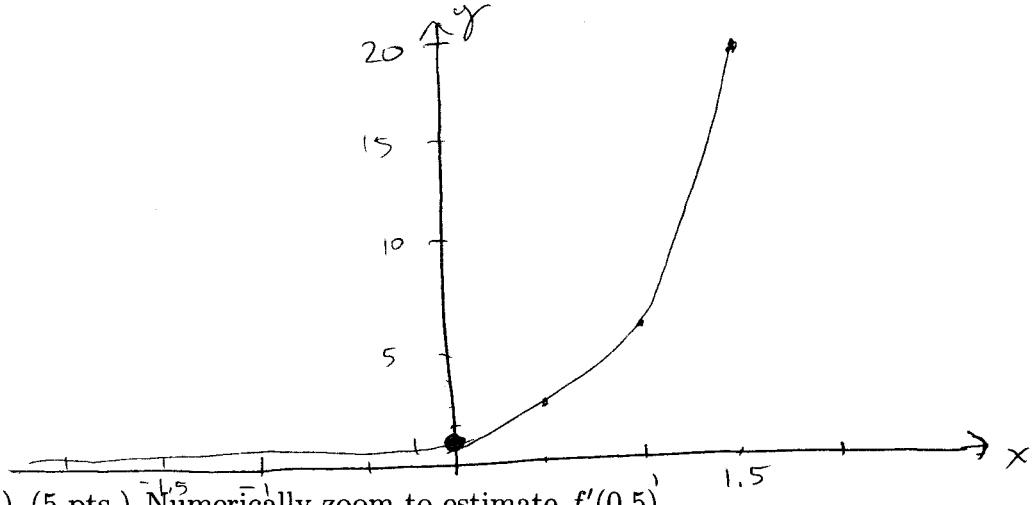
$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

6.) (15 pts.) Consider the function $f(x) = e^{2x}$.

a.) (5 pts.) Sketch $f(x)$ for x-values ranging from -1.5 to 1.5.



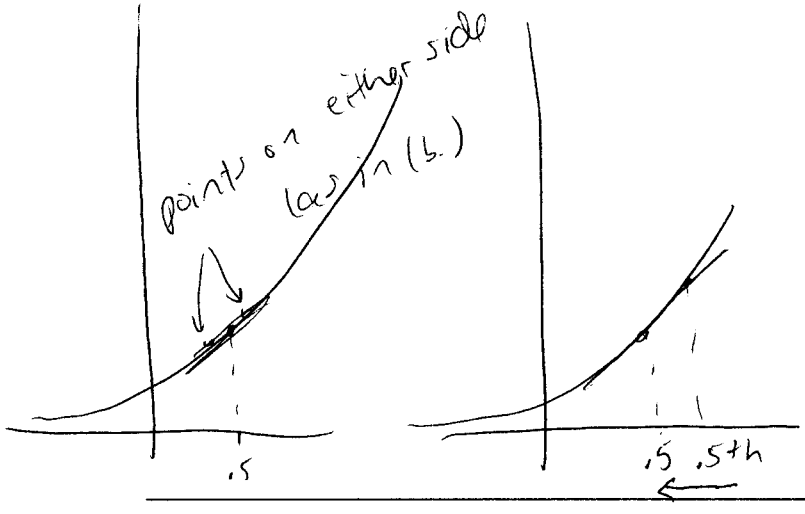
b.) (5 pts.) Numerically zoom to estimate $f'(0.5)$.

x	0.49	0.51
f(x)	2.66	2.77

$$\frac{2.77 - 2.66}{0.51 - 0.49}$$

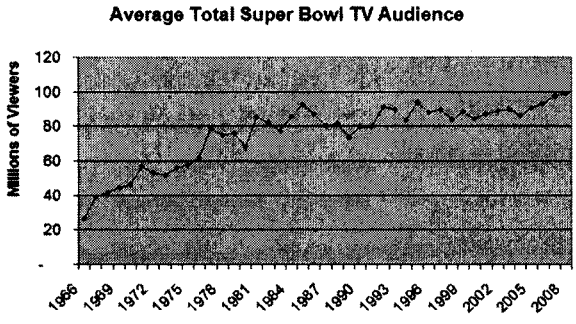
$$\approx 5.5$$

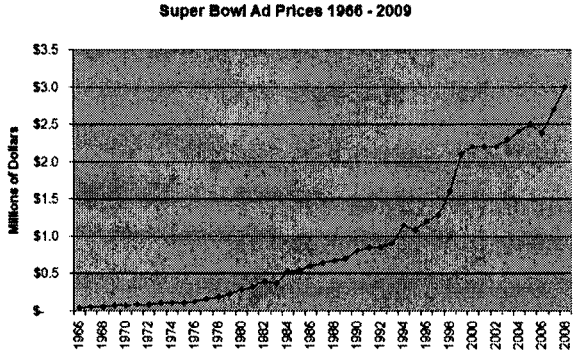
c.) (5 pts.) Explain, referring to your graph, how the idea of numerical zooming leads us to the exact definition of the derivative at a point (such as at the point $x = 0.5$).



If we zoom using, say, $x = 0.5$ and an x-value just a little bigger ($x+h = 0.5+h$) then let h get very close to 0, we get a very good estimate. Calculus lets us bring h all the way to 0.

7.) (15 pts.) The following graphs come from cbsnews.com, accessed February 8, 2012. Use the graphs to respond to the questions below.





a.) (5 pts.) In what year does the change in audience size seem to be the greatest? Estimate the slope of the audience size curve for that year. Make clear how you are estimating.

Those may vary, but name the years and compute $\frac{\text{rise}}{\text{run}}$.

b.) (5 pts.) In what year does the change in ad prices seem to be the greatest? Estimate the slope of the ad price curve for that year. Make clear how you are estimating.

c.) (5 pts.) Across *most* years, what concavity does the audience curve seem to have? Also across *most* years, what concavity does the ad price curve seem to have?

Audience size: down

Ad price: up

BONUS: Based on the graphs, and perhaps on your responses above, what can you conclude about the cost of Super Bowl ads compared with audience size, over the years since the first Super Bowl? (You can write on the back of this page, if you wish.)