TEST 1

Math 105
2/10/12

Name: _____
by writing my name I swear this work is my own

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements. Put a smiley face next to your name for one point.
- This test has 6 problems and is worth 100 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (*10 points*) The position of a car after time t is given by the table of values below.

t (seconds)	0	1	2	3	4	5
s(t) (feet)	0	15	46	72	118	195

a. (*3 pts*) Find the average velocity for the time period beginning when t = 2 and lasting 3 sec.

$$\frac{s(5)-s(2)}{3} = \frac{149}{3} = 49.66 ft/sec$$

b. (*3 pts*) Find the average velocity for the time period beginning when t = 2 and lasting 1 sec.

$$\frac{s(3)-s(2)}{1} = 26 ft/sec$$

c. (*4 pts*) Estimate the instantaneous velocity when t = 2.

$$\frac{s(1)-s(2)}{-1} = 31 ft/sec.$$

Approximately,

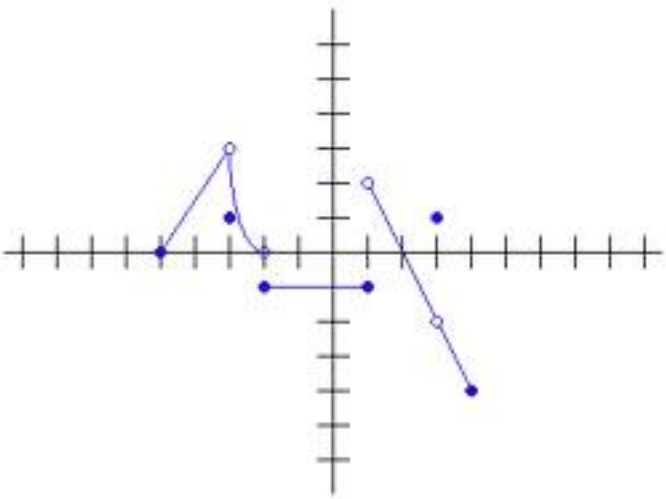
$$28.5 ft/sec.$$

Or use the secant through [1,3]. You will get the same answer.

2. (*10 points*) Use the formal definition of the derivative (ie. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$) to prove that $\frac{d}{dx}(cg(x)) = c\frac{d}{dx}g(x)$ where c is a constant.

$$(cg(x))' = \lim_{h\to 0} \frac{cg(x+h)-cg(x)}{h} = \lim_{h\to 0} c\frac{g(x+h)-g(x)}{h} = c\lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = cg'(x)$$

3. (*19 points*) The graph of $f(x)$ is given. Solving the following (assume the tickmarks occur at 1, 2, etc).



a. (*3 pts*) $\lim_{x\to 1^-} f(x) = -1$

b. (*3 pts*) $\lim_{x\to 1^+} f(x) = 2$

c. (*3 pts*) $f(1) = -1$

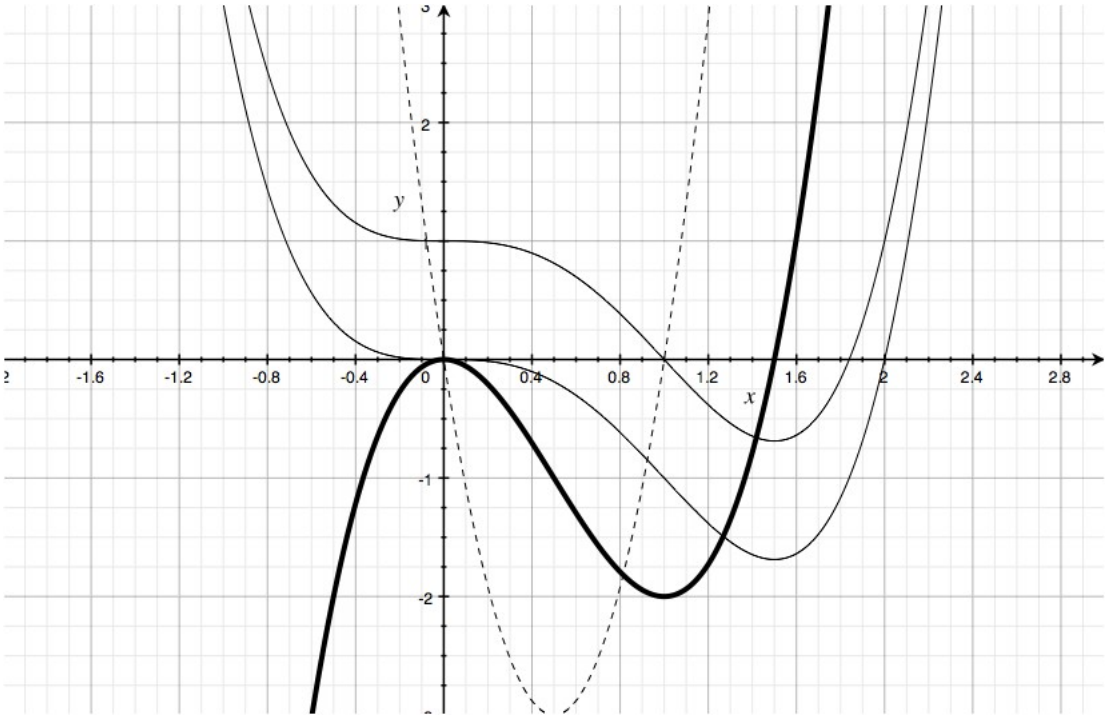
d. (*3 pts*) $\lim_{x\to -2} f(x) = DNE$

e. (*3 pts*) $\lim_{x\to -3} f(x) = 3$

f. (*4 pts*) For what values of x is $f(x)$ NOT continuous?

$f(x)$ is NOT continuous at $x = -3, -2, 1, 3$.

4. (*34 points*) The following is a graph of g' (in bold), NOT g.



a. (*2 pts*) What is $g'(0)$? 0

b. (*2 pts*) What is $g''(0)$? 0

c. (*4 pts*) Draw and label the graph of the g'' on the graph. (dotted)

d. (*12 pts*) On what intervals is g increasing? decreasing? concave up? concave up? concave

$Increasing : (1.5, \infty), Decreasing : (-\infty, 1.5) or (-\infty, 0) \cup (0, 1.5), Concave up : (-\infty, 0) \cup (1, \infty), Concave down : (0, 1)$

e. (*4 pts*) Is $x = 0$ a maximum, minimum, or neither of g? How do you know?

$x = 0$ is neither a min or a max. It is an inflection point. $g''(0) = 0$. Also, the function is increasing before $x = 0$ and increasing after $x = 0$.

f. (*4 pts*) Is $x = 1.5$ a maximum, minimum, or neither of g? How do you know?

$x = 1.5$ is a minimum. $g''(1.5) > 0$ so g is concave up at $x = 1.5$. Also the function is decreasing before and increasing after.

g. (*6 pts*) Sketch and label **2** possible graphs of g on the graph above. (solid lines)

5. (*12 points*) Find the following limits. Use algebra when possible or necessary. If a limit doesn't exist then clearly explain why.

a. (*4 pts*) $\lim_{x\to 1} \frac{7x-7}{3x^2-2}$

$$\lim_{x\to 1} \frac{7x-7}{3x^2-2} = 0$$

b. (*4 pts*) $\lim_{x\to 0} \frac{\sqrt{25-x}-5}{x}$

$$\lim_{x\to 0} \frac{\sqrt{25-x}-5}{x} \cdot \frac{\sqrt{25-x}+5}{\sqrt{25-x}+5} = \lim_{x\to 0} \frac{25-x-25}{x(\sqrt{25-x}+5)} = \lim_{x\to 0} \frac{-1}{x(\sqrt{25-x}+5)} = \frac{-1}{10}.$$

c. (*4 pts*) $\lim_{x\to 3} \frac{|x-3|}{x-3}$

Use a table of values to show that the limit doesn't exist because the right hand and left hand limits are different.

x	$\frac{\lvert x-3\rvert}{x-3}$
2.99	-1
2.999	-1
3.001	1
3.01	1

6. (*14 points*)

a. (*5 pts*) Solve the differential equation $y' = 4x^3 - \frac{6}{x^2} + 2\sqrt{x}$.

$$y = x^4 + \frac{6}{x} + \frac{4}{3}\sqrt{x^3} + C$$

b. (*5 pts*) What is y''?

$$12x^2 + \frac{12}{x^3} + \frac{1}{\sqrt{x}}$$

. **c.** (*4 pts*) If you haven't done so already, write your answers from **a.** and **b.** without fractional or negative exponents.
See **a.** and **b.**.