

Math 105 — First Midterm

February 8, 2013

Name: _____

Instructor: _____ Section: _____

1. **Do not open this exam until you are told to do so.**
2. This exam has 11 pages including this cover AND IS DOUBLE SIDED. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions.
5. Show an appropriate amount of work (including appropriate explanation). Include units in your answer where that is appropriate. Time is of course a consideration, but do not provide no work except when specified.
6. You may use any previously permitted calculator. However, you must state when you use it.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones and hats.
9. Remember that this is a chance to show what you've learned, and that the questions are just prompts.

Problem	Points	Score
1	10	
2	12	
3	12	
4	12	
5	14	
6	16	
7	12	
8	10	
9	02	
Total	100	

1. [10 points] On the axes below sketch a **well-labeled** graph of a continuous function, $f(x)$, which satisfies all of the following properties:

• $g'(x) = 1$ for $0 < x < 1$

• $g'(x) = -1$ for $1 < x < 2$

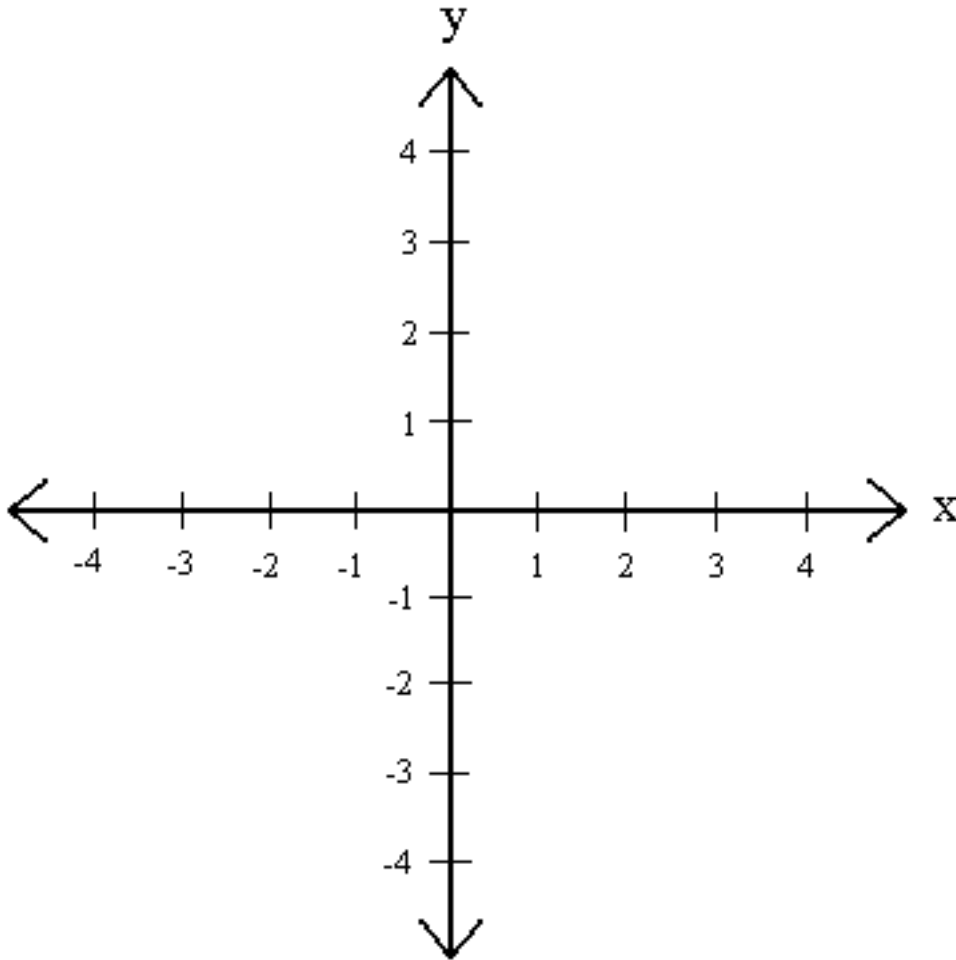
• $g(-1) = -1$

• $g(0) = 0$

• $g'(x) < 0$ for $x > 3$

• $\lim_{x \rightarrow \infty} g(x) = 2$

• g is concave up for $x < 0$



2. [12 points] Sivan takes the Amtrak Downeaster to get down to Boston on the weekends. The train leaves at 2:35 PM, and is a 107 mile trip. If she models her distance in miles down the track by $p(t)$, where t is the number of minutes after the train left, please answer the following questions:

a. [4 points] Translate the equation $f(40) = 25$ to an English sentence.

b. [4 points] In an English sentence, describe what $f'(40) = 1/2$ would imply about $f(39)$.

c. [4 points] If $f'(90) = 0$ what can you conclude for sure about Sivan's location at 4:05? Explain.

3. [12 points] For each problem below, circle **ALL** of the statements that **MUST** be true. The parts are independent of each other. For this problem only, no explanations are required. There is no partial credit.

a. [3 points] If $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) = 1$ then

- $f(x)$ is continuous at 0
- $\lim_{x \rightarrow 0} f(x)$ exists and is equal to 1
- $f'(0) = 1$

b. [3 points] Suppose that $h(t)$ gives the height of a ball, measured in feet above the ground, t seconds after it is thrown. Assume that the derivative of h is $h'(t) = -32t + 96$.

- The ball is at its maximum height 3 seconds after being thrown
- The ball reaches a maximum height of 96 feet from the ground
- The ball hits the ground, and thus has zero change in position, 3 seconds after being thrown

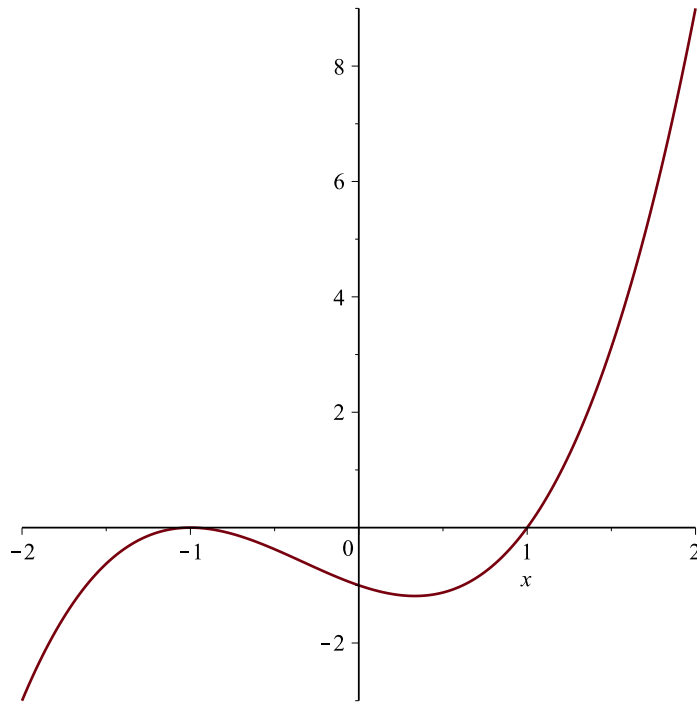
c. [3 points] Suppose that A and B are positive numbers with $A < B$.

- $(\ln(e^A))(\ln(e^B)) = A + B$
- $\ln(A + B) = \ln(A) \ln(B)$
- $\log(A) < \log(B)$
- $\log(B^2 A) = 2 \log(BA)$

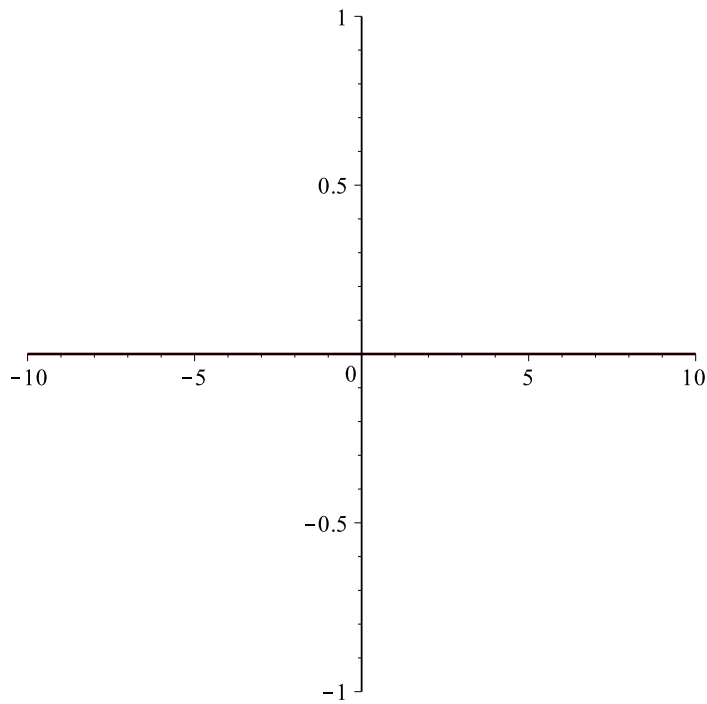
d. [3 points] Suppose that $f'(x) < 0$ for all x .

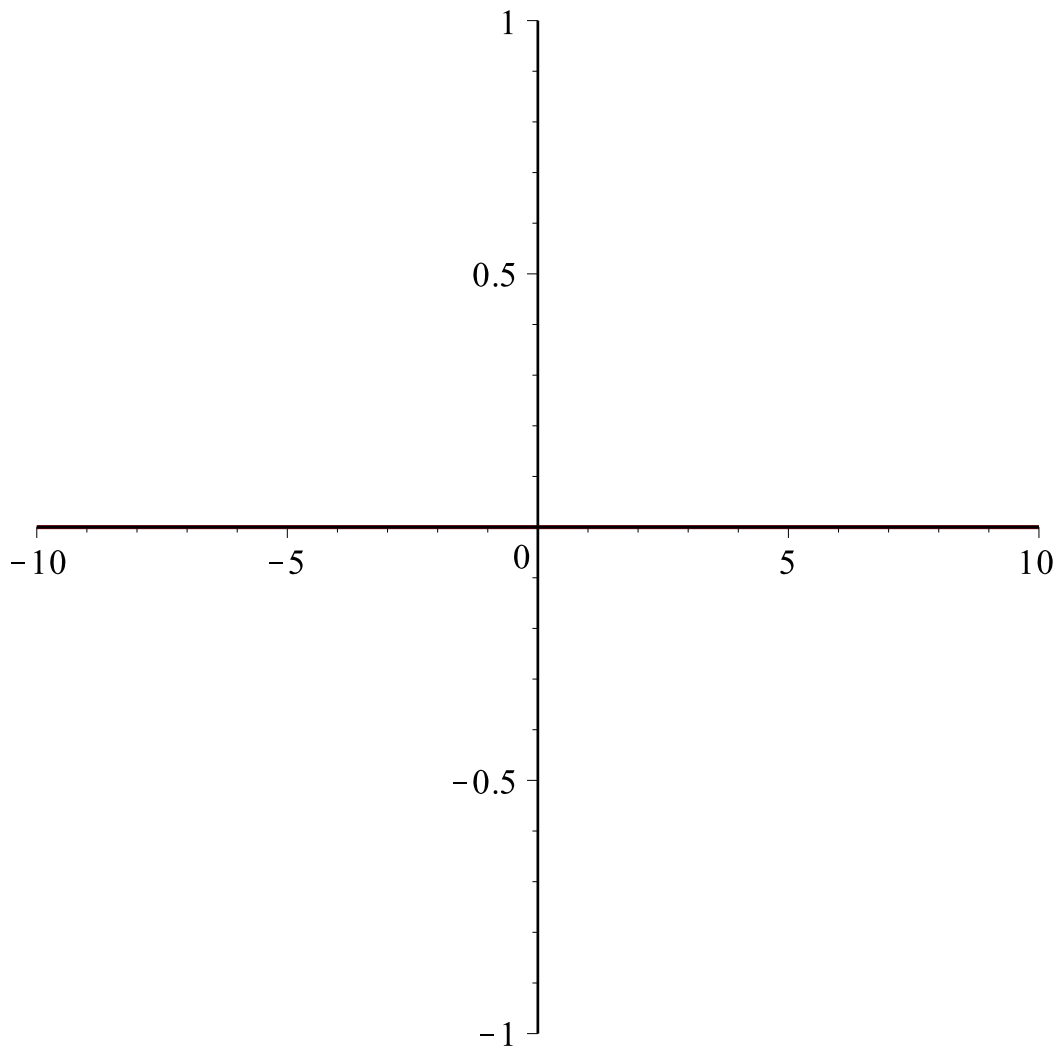
- $f'(x)$ is decreasing for all x
- $f''(x)$ is negative for all x
- $f(x)$ is concave down for all x

4. [12 points] Here is a graph of $f''(x)$:



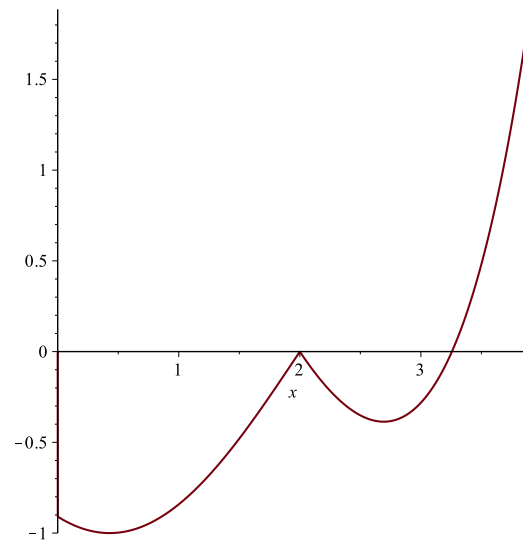
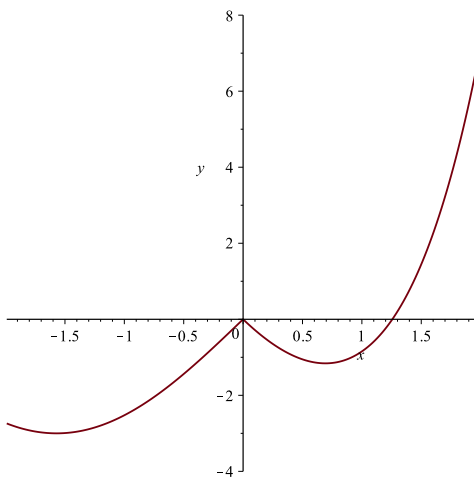
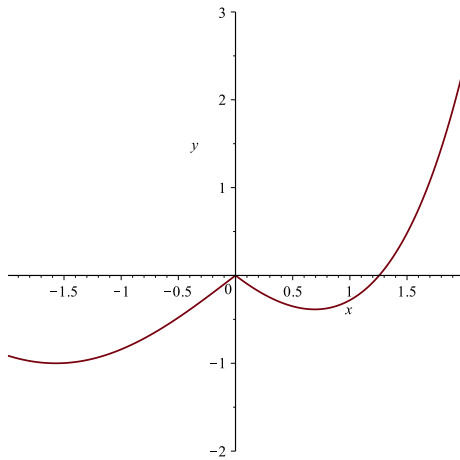
Please graph and label $f'(x)$, $f'''(x)$ on the two graphs below to the best of your abilities.

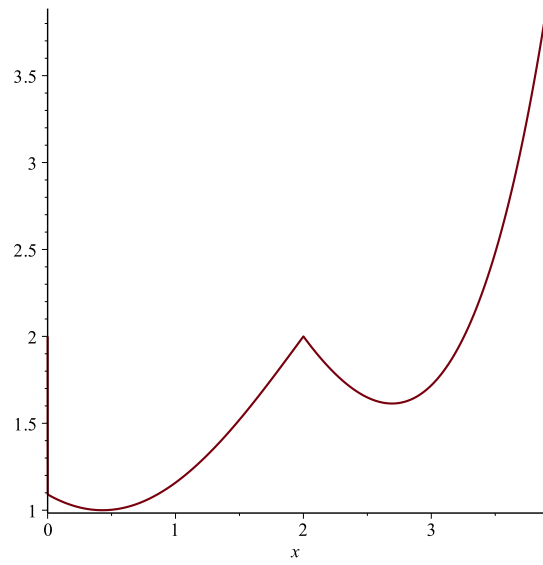
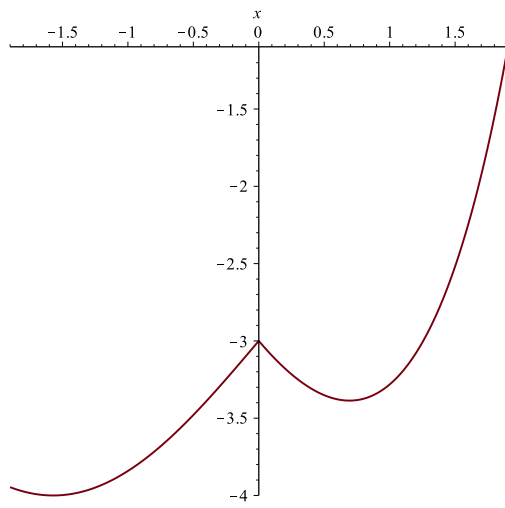




5. [14 points] Let $f(x) = x^x$ for any $x > 0$.
- a. [5 points] Please calculate the average rate of change of $f(x)$ from $x = 1$ to $x = 3$. Show all appropriate work.
- b. [5 points] Please write down, but DO NOT EVALUATE, the limit definition of $f'(1)$.
- c. [4 points] Please make an estimate for your answer in part (b). Explain your work.

6. [16 points] Below is a graph of $g(x)$. Please write the equations for the following 4 graphs in terms of $g(x)$. Please pay close attention to the marked values as the scale is not the same from graph to graph.





7. [12 points] Let $f(x) = 3x^4 + ex^2 + \frac{\pi}{ex^3} - \sqrt{x^3} + e$.
- a. [4 points] Find the derivative of $f(x)$.

b. [4 points] Calculate $\lim_{x \rightarrow 1} f(x)$.

c. [4 points] Find an antiderivative of $f(x)$.

8. [10 points] Let $f(x)$ be a continuous function.
- a. [5 points] Define what it means for a number a to be a critical point of $f(x)$?
- b. [5 points] What problem are critical points good for solving? i.e. Why should we care about critical points?
9. [2 points] How much snow do you think Lewiston will get this weekend?