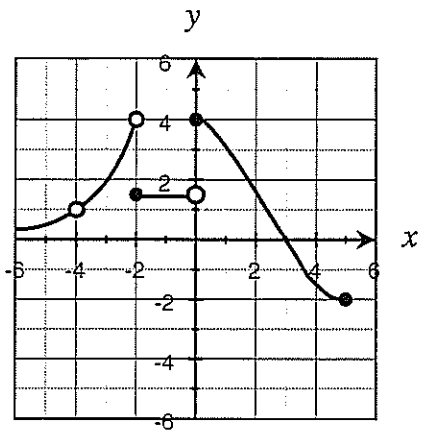
Name: Solutions

Math 105: Winter 2013
Exam 1: February 8

Correct answers accompanied by incorrect or incomplete work will not receive full credit.

1. (4 points each) The graph of $f(x)$ is given. Evaluate the following (assume the tickmarks occur at 1, 2, etc).



(a) $\lim_{x \to 0^-} f(x) = 1.5$

(b) $\lim_{x \to 0^+} f(x) = 4$

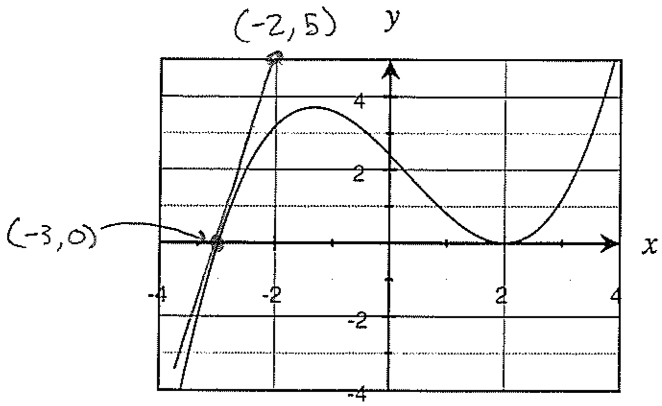
(c) $\lim_{x \to 0} f(x)$ does not exist

(d) $f(0) = 4$

(e) $\lim_{x \to -2} f(x)$ does not exist

(f) $f(-2) = 1.5$

2. (6 points) The graph below is a graph of $f(x)$. Estimate $f'(-3)$.



$f'(3) \approx \frac{5-0}{-2--3} = 5$

3. (6 points) Let $g(x) = \tan x$. Use secant line(s) to numerically estimate $g'(\pi/3)$. (Make sure your calculator is in RADIAN mode.)

$\pi/3 \approx 1.0472$

$g(\pi/3) \approx 1.7321$

$g(1) \approx 1.5574$

$g'(\pi/3) \approx \frac{1.7321 - 1.5574}{1.0472 - 1} \approx 3.7013$

4. (6 points) Let $h(x) = 3x^2 + \frac{6}{x^3} - 4\sqrt{x^3} + 7x^{-2} + 14$. Calculate $h'(4)$.

$h(x) = 3x^2 + 6x^{-3} - 4x^{3/2} + 7x^{-2} + 14$

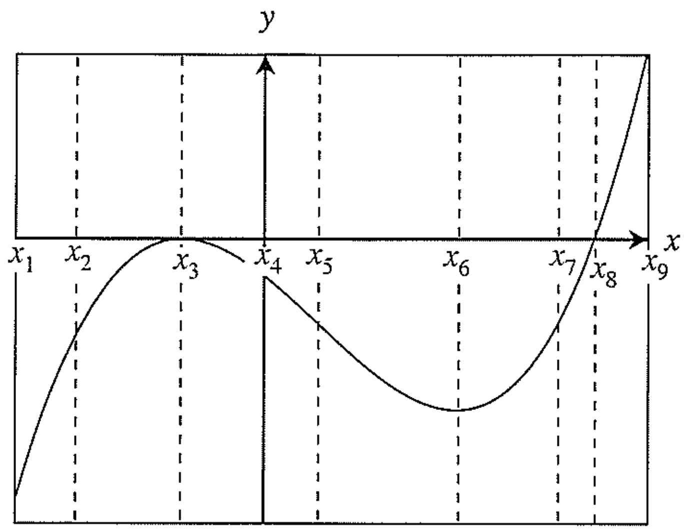
$h'(x) = 6x - 18x^{-4} - 6x^{1/2} - 14x^{-3}$

$h'(4) = 6(4) - 18(4^{-4}) - 6\sqrt{4} - 14(4^{-3}) \approx 11.7109$

5. (4 points) Let $U(t)$ be the number of people unemployed in a country t months after the election of a new president. What does the statement $U'(20) = -10,000$ mean in this context? Include units in your answer.

The number of people unemployed 20 months after the election is decreasing at a rate of 10,000 people per month.

6. (5 points each) The graph below is a graph of $f(x)$.



Let $F(x)$ be an antiderivative of $f(x)$. $F' = f$

(a) For what value(s) of x (if any) does F have a local maximum? Explain your answer.

Stationary points at $F'(x) = f(x) = 0$ so $x = x_3, x = x_8$ are stationary points

$F'(x) = f(x)$: $-$ (x_3) $-$ (x_8) $+$

$F(x)$: $\searrow$ $\searrow$ $\nearrow$

So by the first derivative test F has no local max.

(b) For what value(s) of x (if any) does F have a local minimum? Explain your answer.

As reasoned in (a) F has a local min at $x = x_8$

(c) For what value(s) of x (if any) does F have an inflection point? Explain your answer.

$F(x)$ has inflection points when $F'(x) = f(x)$ has local extrema. So F has inflection points at $x = x_3$ and $x = x_6$.

7. (5 points each) Suppose $g'(w) = \sqrt{w} - 3$. JUSTIFY your answer to each of the following questions WITHOUT GRAPHING g'.

(a) What is the natural domain of $g'(w)$?

$[0, \infty)$ b/c we can't take the square root of a negative number.

(b) Is -3 in the range of $g'(w)$?

yes b/c. $g'(0) = \sqrt{0} - 3 = -3$

so -3 is a possible output

(c) Is g concave up at $w = 4$? $g' = w^{1/2} - 3$

$\boxed{\text{Yes}}$ $g'' = \frac{1}{2} w^{-1/2}$ $g''(4) = \frac{1}{2}(4)^{-1/2} = \frac{1}{4}$

which is positive. so g is concave up

(d) g has a stationary point at $w = 9$. On g, is $w = 9$ a local maximum, local minimum, or neither?

we'll use the second derivative test.

From above $g''(w) = \frac{1}{2\sqrt{w}}$. So $g''(9) = \frac{1}{2\sqrt{9}}$

$= \frac{1}{6}$

So g is concave up @ $w=9$, so g has a local min at $w = 9$.

8. (5 points) Let $h(x) = 3x^2 + \frac{6}{x^3} - 4\sqrt{x^3} + 7x^{-2} + 14$. Find an antiderivative of h.

$h(x) = 3x^2 + 6x^{-3} - 4x^{3/2} + 7x^{-2} + 14$

$\text{antiD} = H(x) = x^3 + \frac{6x^{-2}}{-2} - \frac{4x^{5/2}}{5/2} + \frac{7x^{-1}}{-1} + 14x + C$

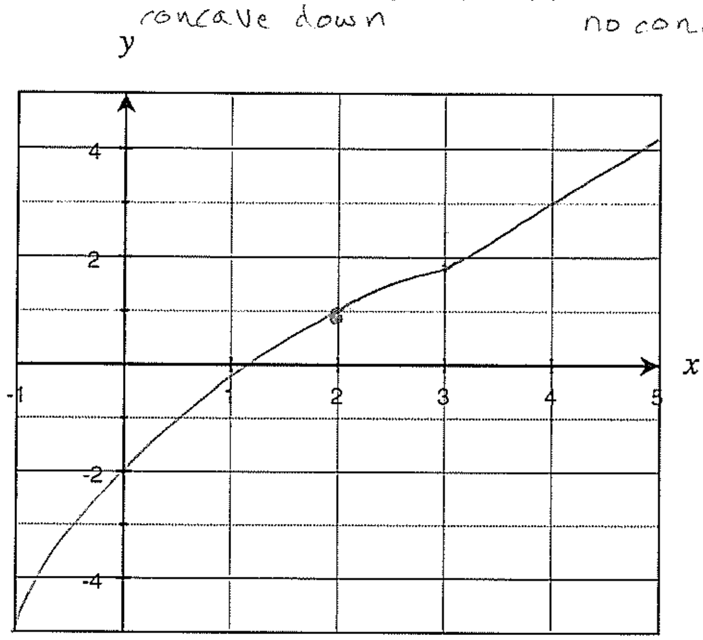
$H(x) = x^3 - 3x^{-2} - \frac{8}{5}x^{5/2} - 7x^{-1} + 14x + C$

9. (6 points) Sketch the graph of a *continuous* function $g(x)$ over the interval $[-1, 5]$ that has the following properties:

- $g(2) = 1$
- $g'(x) > 0$ on the interval $[-1, 3)$, $g'(3)$ does not exist, and $g'(x) > 0$ on the interval $(3, 5]$.

  increasing kink increasing
- $g''(x) < 0$ on the interval $[-1, 3)$, $g''(x) = 0$ on the interval $[3, 5]$.

  concave down no concavity $\Rightarrow g =$ line on $[3, 5]$



10. (5 points) Let $f(x) = \dfrac{3}{x+2}$. Fill in all of the empty spaces in following equation.

one possible answer is below

$$f'(7) = \lim_{\boxed{h \to 0}} \frac{f\left(\boxed{7+h}\right) - f\left(\boxed{7}\right)}{\boxed{h}}$$

$$= \lim_{\boxed{h \to 0}} \frac{\left(\boxed{\frac{3}{7+h+2}}\right) - \left(\boxed{\frac{3}{7+2}}\right)}{\boxed{h}} \quad (f \text{ should NOT appear in this line})$$

11. (3 points) How many inches of snow do you think Lewiston will get this weekend?