Math 105: Review for Exam I
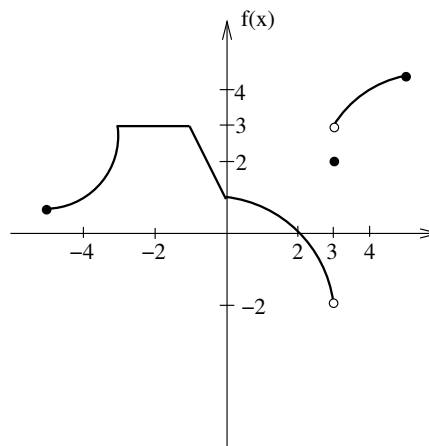
1. Let $f(x) = 3 + \sqrt{x+5}$.

    (a) What is the natural domain of f?

    (b) What is the range of f?

2. For the graph of f shown, answer the following.

    (a) Evaluate the following.

        i. $f'(-2)$

        ii. $f(3)$

        iii. $\lim\limits_{x \to 3^-} f(x)$

        iv. $\lim\limits_{x \to 3^+} f(x)$

        v. $\lim\limits_{x \to 3} f(x)$

        vi. $\lim\limits_{x \to 2} f(x)$

    (b) Where is f discontinuous?

    (c) Where does f' fail to exist?

3. Let $f(x) = 3x^2 - 2x$.

    (a) Compute the average rate of change of f on the interval $[2, 2.1]$.

    (b) Using the limit definition of the derivative, find $f'(x)$.

   (c) Find the equation of the tangent line to f at $x = 2$.

    (d) How would the derivative of $g(x) = f(x) + 5$ compare to $f'(x)$?

    (e) How would the derivative of $h(x) = 5f(x)$ compare to $f'(x)$?

4. Fill in the table showing the graphical relationships between f, f', and f''.
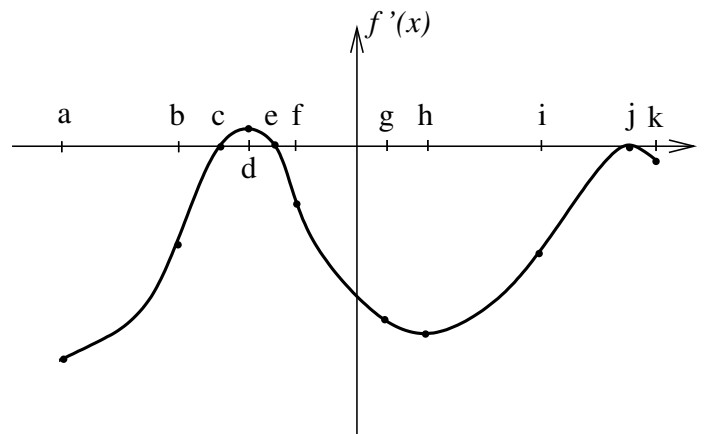
f	positive	negative	increasing	decreasing	concave up	concave down
f'						
f''						

5. Given the graph of f, sketch a graph of f' and a graph of F, an antiderivative of f such that $F(0) = -2$.
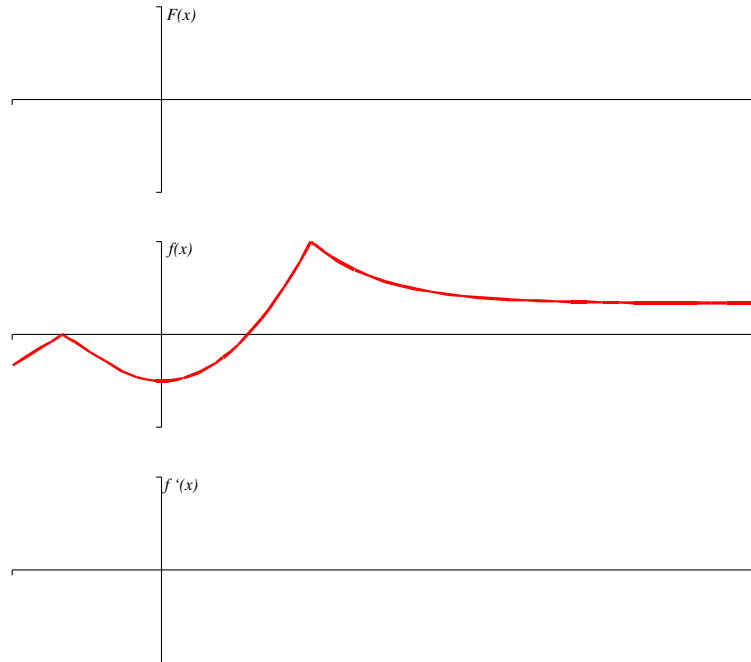
6. Shown below is a graph of f' on its entire domain. The graph is NOT f.

At which x-value(s)

   (a) does f have a stationary point?

   (b) does f have a local max?

   (c) does f have a local min?

   (d) does f' have a stationary point?

   (e) does f' have a local max?

   (f) does f' have a local min?

   (g) is f greatest?

   (h) is f least?

   (i) is f' greatest?

   (j) is f' least?

   (k) is f'' greatest?

   (l) is f'' least?

On what interval(s) is

   (a) f increasing?

(b) f decreasing?

(c) f' increasing?

(d) f' decreasing?

(e) f concave up?

(f) f concave down?

7. Suppose that $T(t)$ gives the temperature in Lewiston as a function of time. In each of the following situations, determine if the signs of T, T', and T'' are positive, negative, zero, or unknown.

   (a) The temperature is 60 degrees and falling steadily.

   (b) The temperature is rising more and more slowly.

   (c) The temperature is -5 degrees and rising.

8. The table below gives some values for a function $f(x)$ whose derivative exists at all x.

x	0.8	0.9	1.0	1.1	1.2
$f(x)$	5.0	6.2	7.3	8.2	9.0

   (a) Estimate $f'(1.05)$.

   (b) Based on the data, is $f''(1.0)$ positive or negative?

9. Find the derivatives of the following.

   (a) $y = 2 + 3x + x^4 + 5x^6$

   (b) $y = \sqrt[6]{x} + \dfrac{1}{x^6} + \dfrac{x}{6} + \dfrac{6}{x} + \dfrac{\pi}{6} + 6^{1/2} + \sqrt{6x^{1/6}}$

10. Find antiderivatives of the following.

   (a) $y = \pi + 3x^2$

   (b) $y = 4x^5 - \dfrac{1}{x^6}$

11. Is $y = 5x^3$ a solution to the differential equation $xy' - 3y = 0$?

12. Solve the IVP (initial value problem) $1 = x^3 - y'(x)$ if $y(2) = 13$.