**Math 205 A & B** Quiz 03 ver01 page 1 02/07/2014 Name *Suggested solutions*

**1.** Let $M = \begin{array}{c} \begin{array}{cccc} \vec{m}_1 & \vec{m}_2 & \vec{m}_3 & \vec{m}_4 \end{array} \\ \begin{bmatrix} 5 & 3 & 3 & 21 \\ 2 & 1 & 0 & 6 \\ 7 & 3 & 3 & 27 \end{bmatrix} \end{array}$; then its RREF is $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$. Label the columns of $M$ as $\mathbf{m}_1$, $\mathbf{m}_2$, $\mathbf{m}_3$, and $\mathbf{m}_4$.

1A.) Use the RREF of $M$ to express $\mathbf{m}_4$ as a linear combination of the other columns and verify your LC "adds up" to $\mathbf{m}_4$, or explain correctly why no such LC can be found.

We're given that the solutions of $M\vec{x} = \vec{0}$

are $\begin{cases} x_1 = -3x_4 \\ x_2 = 0 \\ x_3 = -2x_4 \\ x_4 \text{ is free} \end{cases}$ so choose $x_4 = 1$ (say)

then $-3\vec{m}_1 + 0\vec{m}_2 - 2\vec{m}_3 + 1\vec{m}_4 = \vec{0}$,

or, $\boxed{\vec{m}_4 = 3\vec{m}_1 + 2\vec{m}_3}$

Let's CHECK: $\begin{bmatrix} 21 \\ 6 \\ 27 \end{bmatrix} \stackrel{?}{=} 3\begin{bmatrix} 5 \\ 2 \\ 7 \end{bmatrix} + 2\begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 15 \\ 6 \\ 21 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 21 \\ 6 \\ 27 \end{bmatrix} \checkmark$

NOTE WELL: It's <u>WRONG</u> to label the columns of the RREF as $\vec{m}_1, \dots, \vec{m}_4$; those columns are <u>NO LONGER</u> the original vectors $\vec{m}_1, \dots, \vec{m}_4$ of $M$.

1B.) Use the RREF of $M$ to express $\mathbf{m}_2$ as a linear combination of the other columns and verify your LC "adds up" to $\mathbf{m}_2$, or explain correctly why no such LC can be found.

$\vec{m}_2$ can NOT be expressed as such a L.C. For suppose it <u>is</u>, i.e.

Suppose $\vec{m}_2 = x_1\vec{m}_1 + x_3\vec{m}_3 + x_4\vec{m}_4$ for some scalars $x_1, x_3, x_4$.

then $\vec{0} = x_1\vec{m}_1 + (-1)\vec{m}_2 + x_3\vec{m}_3 + x_4\vec{m}_4$, so we have a sol'n of $M\vec{x} = \vec{0}$ in which the weight of $\vec{m}_2$ is not $0$, a contradiction, since in 1A, we showed that $x_2 = 0$ in <u>EVERY</u> sol'n of $M\vec{x} = \vec{0}$.

**2.** Suppose that $T$ is a transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$. There are two conditions that $T$ must satisfy in order to be a *linear* transformation. They are:

2a. the equation $\boxed{T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})}$ is true for every pair of vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, and

2b. the equation $\boxed{T(\alpha\vec{w}) = \alpha\, T(\vec{w})}$ is true for any scalar $\alpha \in \mathbb{R}$ and vector $\mathbf{w} \in \mathbb{R}^n$.

2c. Define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 + 4x_2 + 10 \\ x_1 x_2 \end{bmatrix}$. Show that $T$ fails to satisfy the equation in (2a) using the vectors $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$.

First we find $T(\vec{u}+\vec{v}) = T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 6 \\ 5 \end{bmatrix}\right) = T\left(\begin{bmatrix} 7 \\ 7 \end{bmatrix}\right) = \begin{bmatrix} 21+28+10 \\ 49 \end{bmatrix} = \begin{bmatrix} 59 \\ 49 \end{bmatrix}$

next, $T(\vec{u}) + T(\vec{v}) = T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) + T\left(\begin{bmatrix} 6 \\ 5 \end{bmatrix}\right) = \begin{bmatrix} 3+8+10 \\ 2 \end{bmatrix} + \begin{bmatrix} 18+20+10 \\ 30 \end{bmatrix}$

$= \begin{bmatrix} 21 \\ 2 \end{bmatrix} + \begin{bmatrix} 48 \\ 30 \end{bmatrix} = \begin{bmatrix} 69 \\ 32 \end{bmatrix}$

Since these are unequal, $T$ fails to be a LINEAR transformation.