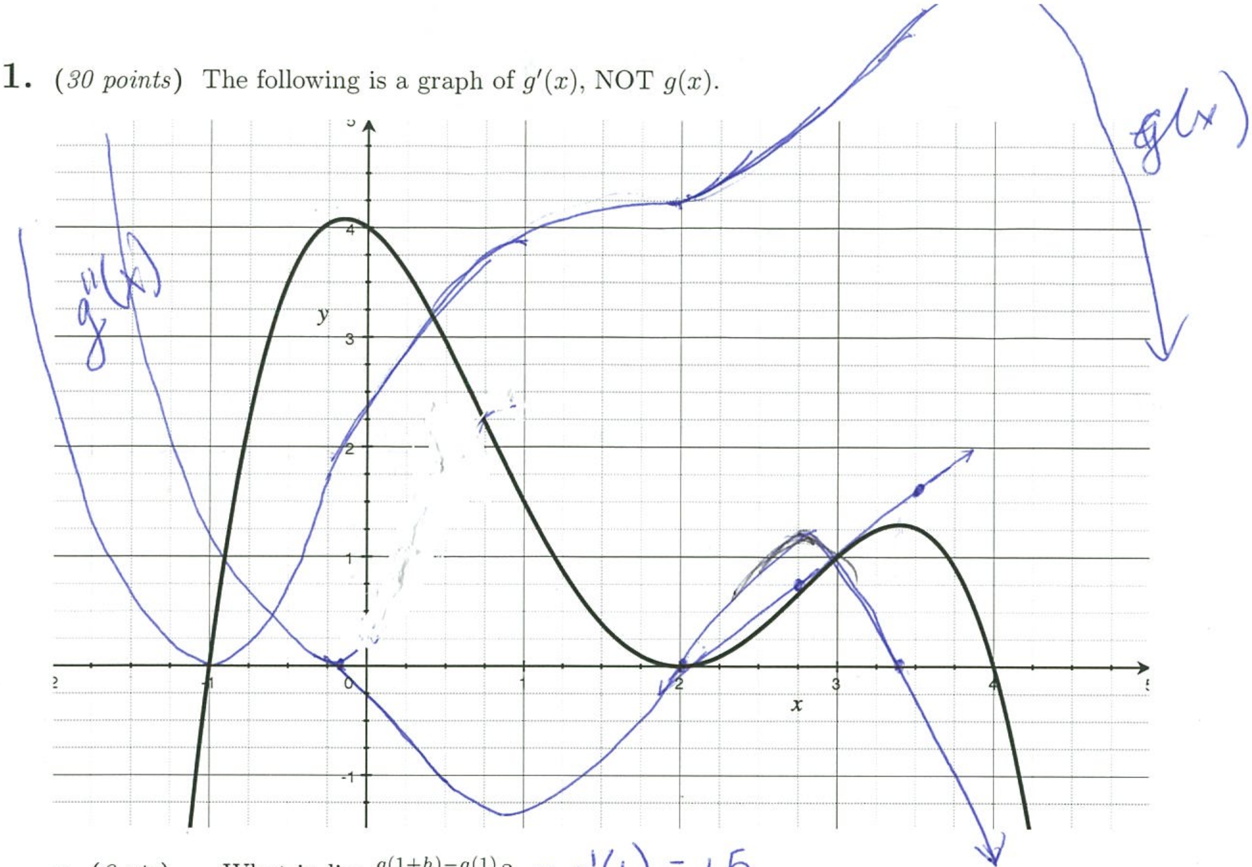
1. (*30 points*) The following is a graph of $g'(x)$, NOT $g(x)$.



a. (*3 pts*) What is $\lim_{h \to 0} \frac{g(1+h)-g(1)}{h}$? $= g'(1) = 1.5$

b. (*4 pts*) Estimate $g''(3)$ by drawing in a tangent line.

$(2.75, .75)$ & $(3.5, 1.6)$

$g''(3) \approx \frac{1.6 - .75}{3.5 - 2.75} = \frac{.85}{.75} = 1.1\overline{3}$

c. (*3 pts*) On what interval(s) is $g(x)$ increasing?

($g'(x)$ positive)

$(-1, 4)$

d. (*3 pts*) On what interval(s) if $g(x)$ concave down?

($g''(x)$ negative or $g'(x)$ decreasing)

$(-.15, 2) \cup (3.4, \infty)$

e. (*6 pts*) Where does $g(x)$ have stationary points? Classify each as max points, min points, or neither.

($g'(x) = 0$)

$x = -1$ min, $x = 2$ neither, $x = 4$ max

f. (*3 pts*) Where does $g(x)$ have inflection points? ($g''(x) = 0$)

$x = -.15, 2, 3.4$

g. (*4 pts*) Sketch and label the graph of $g''(x)$ on the graph above.

h. (*4 pts*) Sketch and label a possible graph of $g(x)$ on the graph above.

2. *(15 points)*
a. *(10 pts)* Use the limit definition of the derivative to find $f'(x)$ if $f(x) = 3x^2 + 7x + 1$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)^2 + 7(x+h) + 1 - (3x^2 + 7x + 1)}{h}$$

$$= \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 + 7x + 7h + 1 - 3x^2 - 7x - 1}{h} = \lim_{h\to 0} \frac{h(6x+3h+7)}{h}$$

$$= \lim_{h\to 0} 6x + 3h + 7 = 6x + 7$$

b. *(5 pts)* Find the equation of the line tangent to $f(x) = 3x^2 + 7x + 1$ for $x = -1$.

$f'(x) = 6x + 7, \quad f'(-1) = -6 + 7 = 1 \qquad f(-1) = 3 - 7 + 1 = -3$

$y + 3 = x + 1$

$\boxed{y = x - 2}$

3. *(9 points)* Determine the following using the power rule and sum/difference rule. Rewrite the final answer to remove all negative and fractional exponents.

$$\frac{d}{dx}\left(2\sqrt[3]{x^5} + \frac{3}{5x^2} - 2x^{-3} + 5x + 2^3\right)$$

$$\frac{d}{dx}\left(2x^{5/3} + \frac{3}{5}x^{-2} - 2x^{-3} + 5x + 2^3\right)$$

$$= \frac{10}{3}x^{2/3} - \frac{6}{5}x^{-3} + 6x^{-4} + 5$$

$$= \frac{10\sqrt[3]{x^2}}{3} - \frac{6}{5x^3} + \frac{6}{x^4} + 5$$

4. (*18 points*) Using derivatives and antiderivatives

a. (*7 pts*) Find constants a and b such that the polynomial $p(x) = x^3 + ax + b$ is increasing for $x < 1$ and decreasing for $x > 1$ but $p(1) = -3$.

$x = 1$ is a max

$p'(x) = 3x^2 + a$

$p''(x) = 6x$ (note $p''(1) > 0$ so $p(x)$ is cc. $\uparrow$ $x=1$ cannot be a max)

The function doesn't exist.

but if you continued.

$p'(1) = 0$ so $3 + a = 0 \Rightarrow a = -3$

$p(x) = x^3 - 3x + b$

$-3 = p(1) = 1 - 3 + b = -2 + b \Rightarrow b = -1$

$p(x) = x^3 - 3x - 1$

b. (*5 pts*) Is $y = 2x^5$ a solution to the IVP $y' = \frac{10y}{x}$, $y(1) = 2$. Justify your answer.

$y' = 10x^4$, $\frac{10y}{x} = \frac{10(2x^5)}{x} = 20x^4$

NO

$10x^4 \neq 20x^4$

c. (*6 pts*) Solve the following differential equation with initial conditions $y(0) = 2$ and $y'(0) = 4$

$$y'' = 2x$$

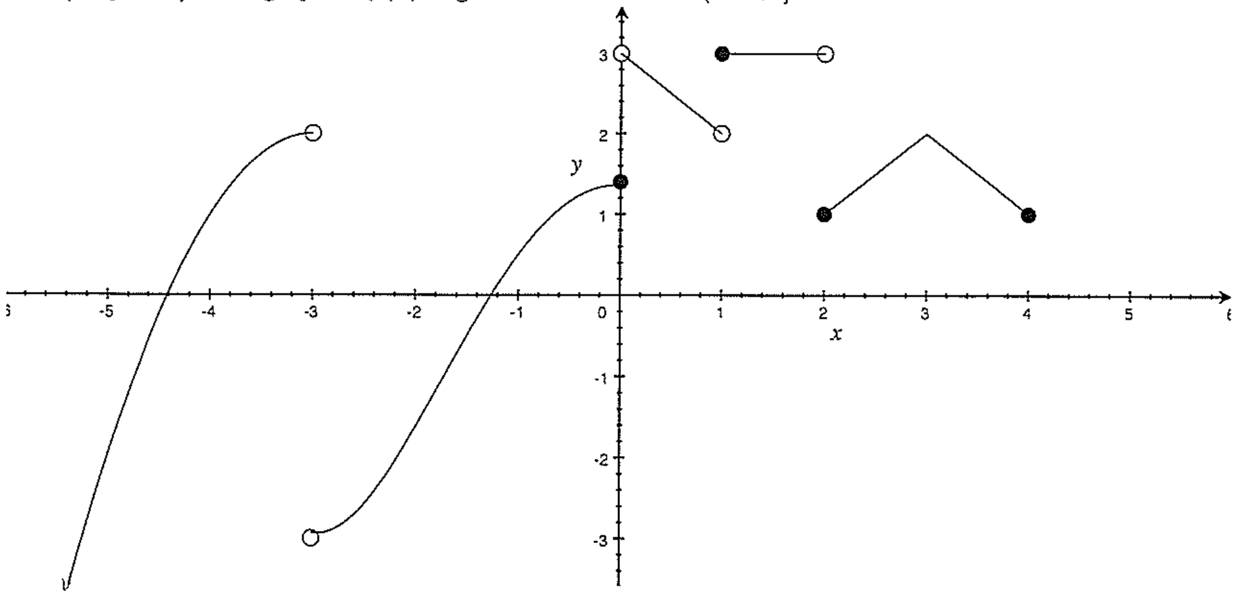
$y' = x^2 + C$, $y'(0) = 4$

$\Rightarrow y' = x^2 + 4$

$y = \frac{x^3}{3} + 4x + C$, $y(0) = 2$

$\Rightarrow y = \frac{x^3}{3} + 4x + 2$

5. (*19 points*) The graph of $f(x)$ is given on the interval $(-\infty, 4]$.



a. (*2 pts*) $\lim_{x\to -3^-} f(x) =$ 2

b. (*2 pts*) $\lim_{x\to -3^+} f(x) =$ -3

c. (*2 pts*) $f(1) =$ 3

d. (*2 pts*) $\lim_{x\to 1} f(x) =$ DNE

e. (*2 pts*) $\lim_{x\to 3} f(x) =$ 2

f. (*3 pts*) Where is the function not continuous (which x-values)?

$x = -3, 0, 1, 2$

g. (*2 pts*) Where is the function not differentiable (which x-values)?

$x = -3, 0, 1, 2, \underline{3}$

h. (*4 pts*) What is the range of the function?

$(-\infty, 3]$

6. (*8 points*) Solve the following limit in two ways. One of the ways must be through algebraic manipulation. Show your work for both ways.

$$\lim_{x\to 0} \frac{\frac{2}{1+x} - 2}{x}$$

1) $\lim_{x\to 0} \dfrac{\frac{2}{1+x} - \frac{2(1+x)}{(1+x)}}{x} = \lim_{x\to 0} \dfrac{2-2-2x}{(1+x)x} = \lim_{x\to 0} \dfrac{-2}{(1+x)} = \boxed{-2}$

2) Table

x	$\left(\frac{2}{(1+x)} - 2\right)/x$
$-.1$	$-2.\overline{2}$
$-.01$	-2.02
$.01$	$-1.\overline{98}$
$.1$	$-1.\overline{81}$

$\to \boxed{-2}$