MATH106A CALCULUS II - PROF. P. WONG

EXAM I - FEBRUARY 6, 2015

NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	**Maximum Score**	**Your Score**
1.	20	
2.	20	
3.	20	
4.	20	
5.	20	
Total	100	

1.(10 pts.)(a) Evaluate the indefinite integral (be sure to show all your work)

$$\int \frac{4z\ dz}{\sqrt{2z^2+1}}.$$

(10 pts.) (b) Find the **exact value** of the definite integral (be sure to show all your work)

$$\int_0^{\pi} 3\cos^2 x \sin x\ dx.$$

2. (12 pts.) (a) Consider the region A bounded by the curve $y = 2 - x^2$ and the line $y = -x$. Find the **exact area** of the region A.

(8 pts.)(b) Find the (**exact**) length of the curve $y = \frac{1}{3}(x^2 + 2)^{3/2}$ between $x = 0$ and $x = 3$.

3. (12 pts.) Consider a function h on the interval $[1, 4]$.

x	1	1.5	2	2.5	3	3.5	4
$h(x)$	-1	1	3	2	0	-3	-2

Find T_6, M_3 using the trapezoid rule and the mid-point rule respectively for estimating the definite integral $\int_1^4 h(x)\ dx$.

(8 pts.)(b) Recall that the error committed by using the left hand sum approximation L_n is less than or equal to $\frac{K_1 \cdot (b-a)^2}{2n}$ where $|f'(x)| \leq K_1$ for some constant K_1 over the interval $[a, b]$. Use this result to give an upper bound for the error committed by L_{10} for

$$I = \int_0^1 e^{-t^2}\ dt.$$

4. Let R be the region in the first quadrant bounded by the curve $x = 3 - y^2$, the line $y = \sqrt{3}$, and the line $x = 3$.

(12 pts.) (a) Find the **exact volume** of the solid obtained from rotating the region R around the y-axis. [Hint: sketch a picture of the region R first.]

(8 pts.) (b) Set up (do not evaluate) a definite integral representing the volume of the solid obtained from rotating the region R around the x-axis.

5. (10 pts.)(a) Consider the initial value problem

$$\frac{dy}{dx} = 2xe^{-y}$$

with $y(0) = 0$. Use the technique of separation of variables to solve the Initial Value Problem.

(10 pts.)(b) A rectangular tank (width 10 ft, length 12 ft, height 20 ft), with its top at ground level, is used to catch runoff water. Assume that the water weighs 62.4 lb/ft^3. How much work does it take to empty the tank by pumping the water back to ground level once the tank is full? [Draw a picture!]